# Purushottam School of Engineering and Technology, Rourkela

# Lectures notes
# On

## MACHINE DESIGN (MET-501)

$5^{th}$ SEM MECHANICAL

Department of Mechanical Engg.

Prepared by:-
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(Lecturer)

# MACHINE DESIGN

| | | | |
|---|---|---|---|
| Course code: | MET-501 | Semester : | 5th |
| Total Period: | 60 | Examination : | 3 hrs (Design data book allowed) |
| Theory periods: | 4 P/W | Class Test: | 20 |
| Maximum marks: | 100 | Teacher's Assessment: | 10 |
| | | End Sem Examination: | 70 |

## Rationale:

Machine design is the art of planning or devising new or improved machines to accomplish specific purposes. Idea of design is helpful in visualizing, specifying and selection of parts and components which constitute a machine. Hence all mechanical engineers should be conversant with the subject.

## Course Objectives:

1. Understanding the behaviours of material and their uses.
2. Understanding the design of various fastening elements and their industrial uses.
3. Understanding the different failures of design elements.
4. Understanding the change of design to accomplish the different field of applications.

| | | | Periods |
|---|---|---|---|
| **1.0** | **Introduction:** | | |
| | 1.1 | Introduction to Machine Design and Classify it. | 8 |
| | 1.2 | State the types of loads. | |
| | 1.3 | Define working stress, yield stress, ultimate stress & factor of safety. | |
| | 1.4 | State mechanical properties of the material. | |
| | 1.5 | State the factors governing the design of machine elements. | |
| | 1.6 | Describe design procedure. | |
| **2.0** | **Design of fastening elements:** | | 14 |
| | 2.1 | State nomenclatures, form of threads & specifications. | |
| | 2.2 | Design of Screw thread (Nut and Bolt) | |
| | 2.3 | State types of welded joints. | |
| | 2.4 | State advantages of welded joints over other joints. | |
| | 2.5 | Determine strength of welded joints for eccentric loads. | |
| | 2.6 | State types of riveted joints. | |
| | 2.7 | Describe failure of riveted joints. | |
| | 2.8 | Determine strength & efficiency of riveted joints. | |
| | 2.9 | Design riveted joints for pressure vessel. | |
| | 2.10 | Solve numerical on Screw thread, Welded Joint and Riveted Joints. | |
| **3.0** | **Design of shafts and Keys:** | | 12 |
| | 3.1 | State function of shafts. | |
| | 3.2 | State materials for shafts. | |
| | 3.3 | Design solid & hollow shafts to transmit a given power at given rpm based on<br>a) Strength: (i) Shear stress, (ii) Combined bending & tension;<br>b) Rigidity: (i) Angle of twist, (ii) Deflection, (iii) Modulus of rigidity | |
| | 3.4 | State standard size of shaft as per I.S. | |
| | 3.5 | State function of keys, types of keys & material of keys. | |

| | | | |
|---|---|---|---|
| | 3.6 | Describe failure of key, effect of key way. | |
| | 3.7 | Design rectangular sunk key considering its failure against shear & crushing. | |
| | 3.8 | Design rectangular sunk key by using empirical relation for given diameter of shaft. | |
| | 3.9 | State specification of parallel key, gib-head key, taper key as per I.S. | |
| | 3.10 | Solve numerical on Design of Shaft and keys. | |
| **4.0** | **Design of belt drivers and pulleys:** | | 14 |
| | 4.1 | State types of belt drives & pulleys. | |
| | 4.2 | State formula for length of open and crossed belt, ratio of driving and driven side tension, centrifugal tension, relation between centrifugal tension and tension on tight side for maximum power transmission. | |
| | 4.3 | Determine belt thickness and width for given permissible stress for open and crossed belt considering centrifugal tension. | |
| | 4.4 | Design a cast iron (C.I) pulley using empirical formula only. | |
| | 4.5 | Solve numerical on design of belt and design of C.I pulley. | |
| **5.0** | **Design a closed coil helical spring:** | | 12 |
| | 5.1 | Materials used for helical spring. | |
| | 5.2 | Standard size spring wire. (SWG). | |
| | 5.3 | Terms used in compression spring. | |
| | 5.4 | Stress in helical spring of a circular wire. | |
| | 5.5 | End connection for helical tension spring. | |
| | 5.6 | Deflection of helical spring of circular wire. | |
| | 5.7 | Eccentric loading of spring. | |
| | 5.8 | Surge in spring. | |
| | 5.9 | Solve numerical on design of spring. | |

**Learning Resources:**

| *Sl. No.* | *Name of Authors* | *Title of the Book* | *Name of the Publisher* |
|---|---|---|---|
| 1 | R.S. Khurmi & J.K. Gupta | A text book of Machine Design | S.Chand |
| 2 | P.C. Sharma & D.K. Aggarwal | A text book of Machine Design | S.K Kataria & Sons |
| 3 | V.B. Bhandari | Design of machine element | TMH |
| 4 | S. Md. Jalaludeen | Design data handbook | Anuradha Publication |

## Introduction

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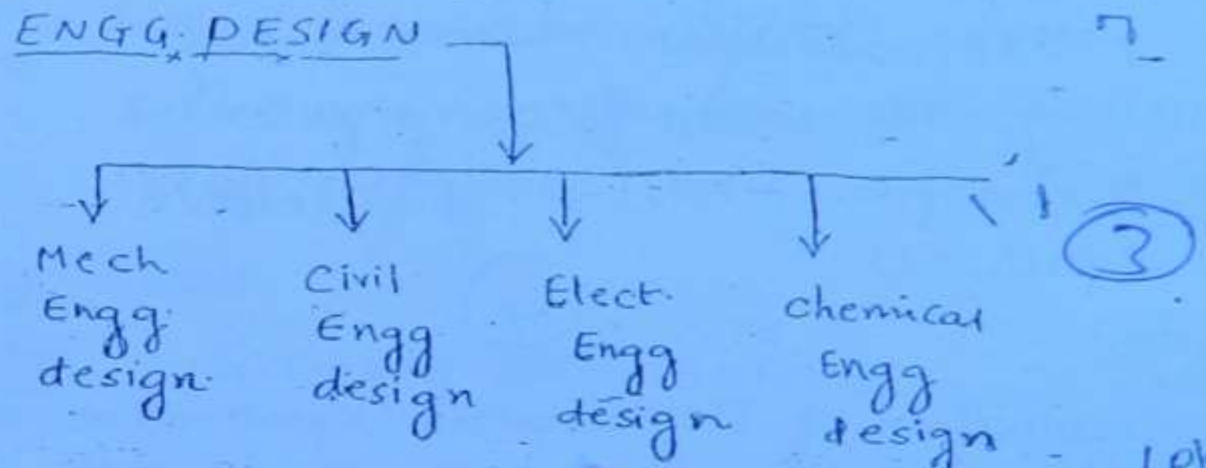


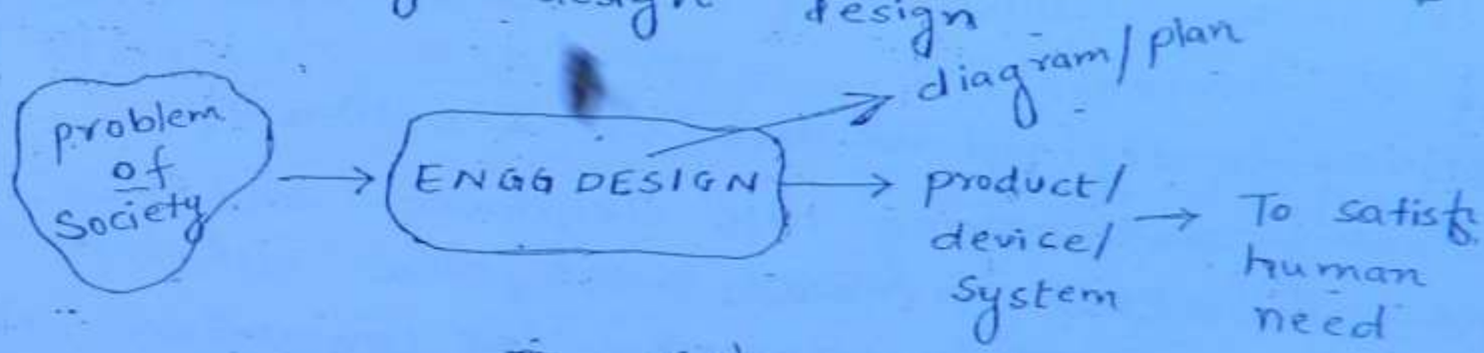


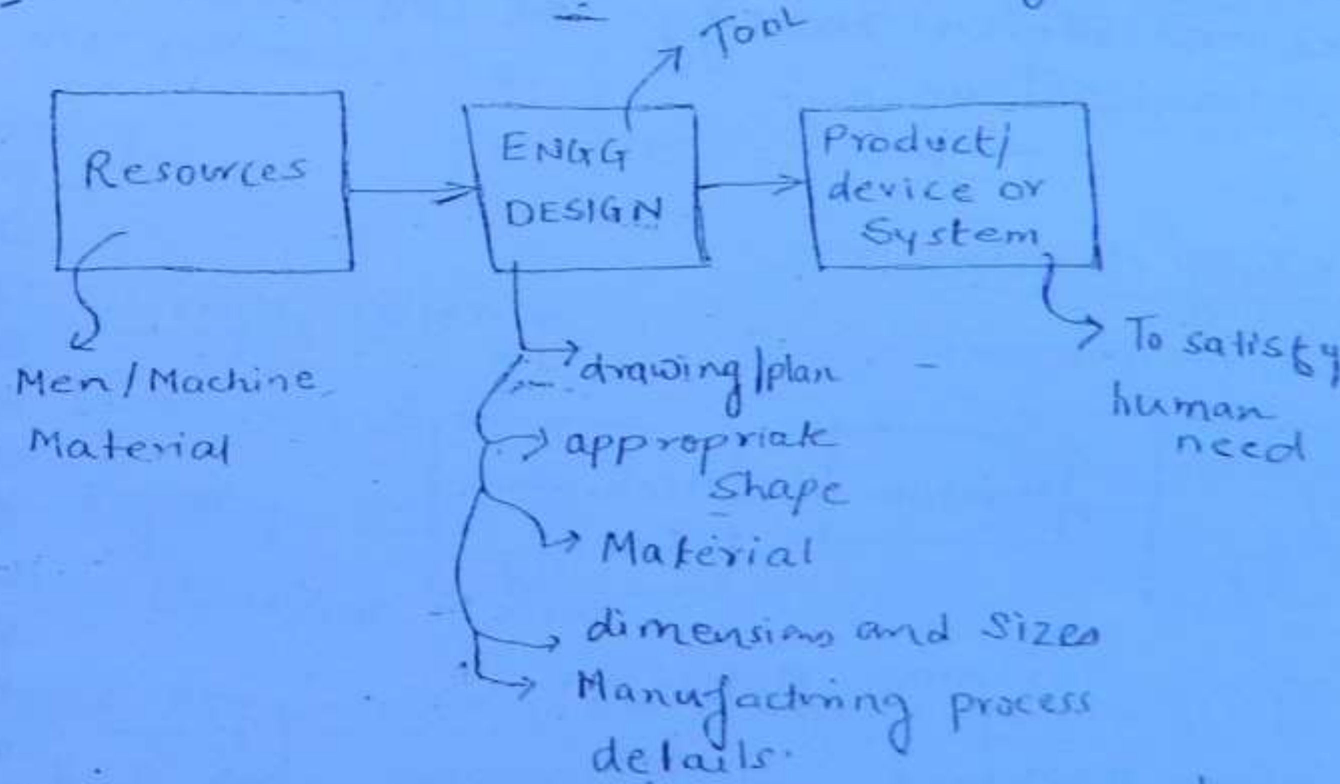


ENGG. Design: It is defined as an Iterative decision making activities to produce a drawing or a plan, to convert resources optimally into a product or device or a system to satisfy the human need.

The ultimate aim of design is to select appropriate shape material, size and nufacturing process details in such a way t the resulting m/c component should rform its given function satisfactorily (ie, ithout any failure).

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## Machine

It is a combination of Mechanisms (Combination of M/c elements).

Machine is defined as the combination of ationary and moving m/c elements and they are assembled in such a way that either produce mechanical energy or convert or tilise mechanical energy

### Types

Generating Machine

eg Prime movers

Some form of energy → Machine → Mechanical Energy

eg Engines, Turbines, Motors

Generating Machines

Mechanical Energy → M/c → Some form of energy

eg. Generators. hand pum

## Selection of appropriate Material

1. List of properties required
2. selecting group of Materials
3. Availability
4. cost
5. selecting a best Material  5

## Friction lining Material

| ⇒ 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| ⇒ Strength ↑ | X | More | Costlier | |
| ⇒ $\mu$ ↑ | Y | | | $\propto$ |
| ⇒ Weer resistence ↑ | Z | | | |
| ⇒ K ↑ | W | less | cheap | |

1. Strength criterion

$$(\sigma_{max})_{ind.} \leq \sigma_{per}$$

2. Rigidity criterion

$$(\delta_{max})_{ind} \leq \delta_{per}$$

## Basic Requirements for a Machine Elements

1. high strength
2. More rigidity
3. high service life
4. less cost
5. More wear resistant

I) POWER-TRANSMISSION SYSTEMS

called as Mechanical power Transmission systems (MPTS)

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i) MPTS

- Flexible PTS (slack drive) $VR = \frac{N2}{N1} \neq C$
  - lt rive
    - V belt drives (smaller CD)
  - chain drive
    - Bush & Roller Chains
    - Silent chains
  - Rope drive
    - fibre Rope drive CD=60m
    - Wire Ropes (D=150m
- Non flexible PTS
  - Gear drives (GD) +ve drive $VR = N2/N1 = C$
    - SPUR GD
    - Helical GD
    - Bevel GD
    - Worm GD (Speed Reducer)

ictors to be considered in the selection of a roper MPTS

Centre distance (C.D)

Shaft layout

power to be transmitted

velocity Ratio.

## Advantage of Flexible PTS

① Larger centre distance

② cost is less

③ Centre distance can be achieved

④ damping capacity is more

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## Disadvantages of flexible PTS

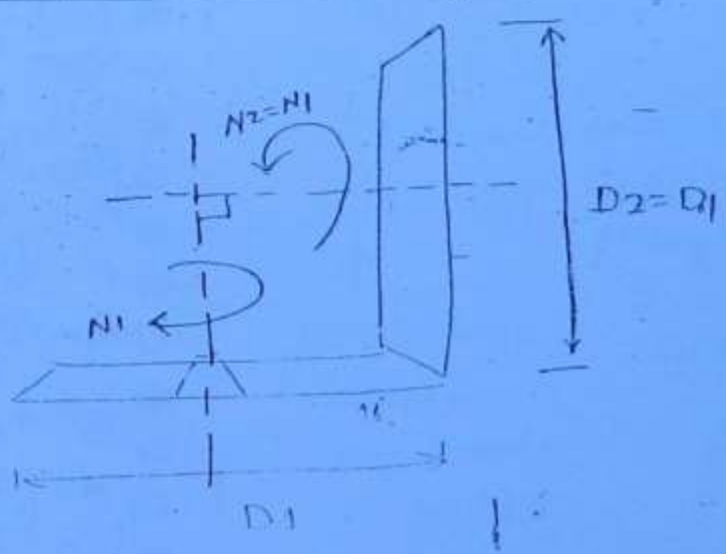
①. velocity ratio is not constant due to slip

② efficiency is less

⑤ Service life is less

## MITRE GEARS

Two equal sized gear mounted on two intersecting perpendicular shafts.




Spur Gear

$F_r$ and $F_T$ → thrust force

$F_a = 0$ (Axial force is zero)

$F_a = F_t \tan\beta$

$\beta = 0 \Rightarrow F_a = 0$

## Steps used in Design of a Machine Element



Specify function of a M/c Element

↓

Determination of loads acting on a M/c element

↓

Selection of an appropriate shape for a Machine element

→ expression for Geometrical properties of the selected shape

↓

Selection of appropriate material for the Machine element

→ properties of that selected Material
eg E, G, YS, US, EL

↓

Selection of Mode of failure

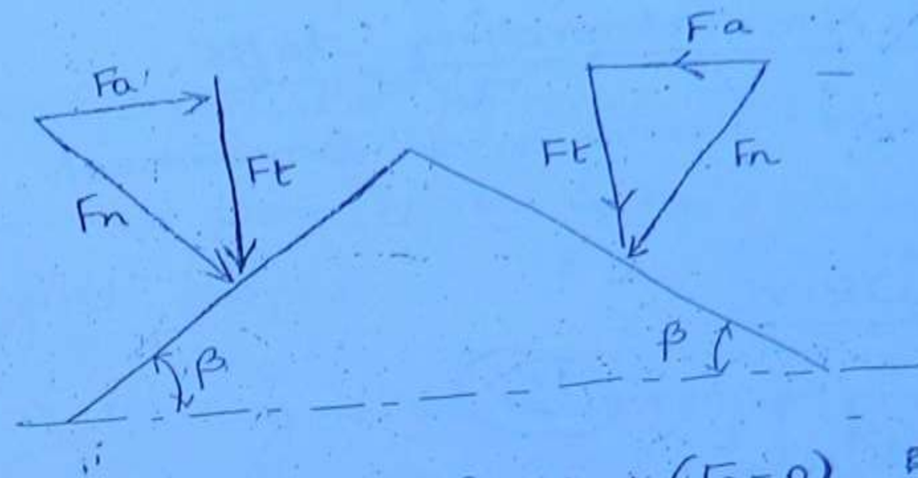
→ failure by Elastic deflection ⇒ Elastic limit
→ failure by yielding ⇒ YS
→ failure by fracture ⇒ U.S

↓

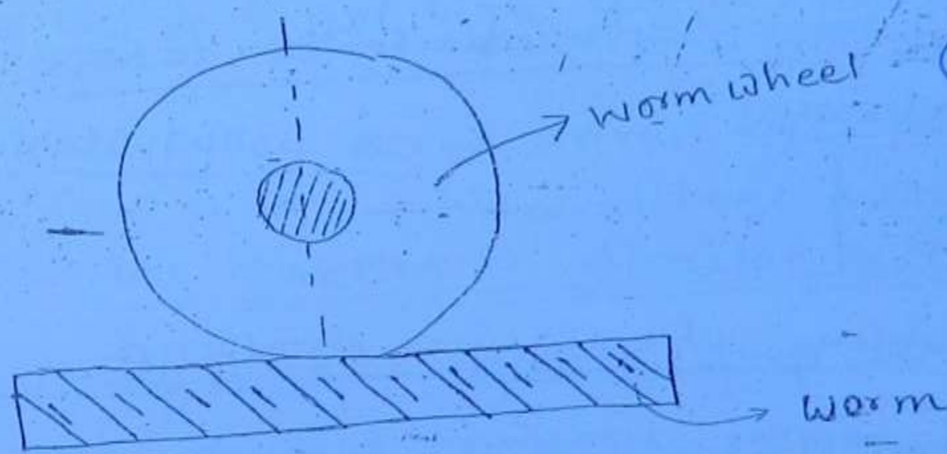
Determination of dimensions by using Strength of Material equations

↓

Preparation of part drawing for the given Machine element




→ axial thrust is eliminated ($F_a = 0$)

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(multi start power screw)

$\boxed{\text{Lead} = np}$

Lead = axial distance travelled by the scre[w] in one rotation

$p$ = pitch
$n$ = no. of starts

$L = p \Rightarrow$ single start power screw

$L = 2p \Rightarrow$ double start power screw

In above case efficiency is less but speed reduction ratio is high

for Non parallel Non Intersecting shafts
eg hypod gear, worm and worm wheel

1 (a) FLAT BELT DRIVE

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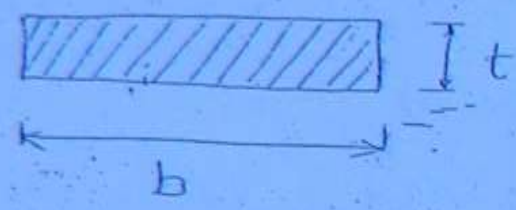

always preferable thin and wider belts to have less bending stresses

as $\boxed{\sigma_b = \frac{E \cdot t}{D}}$

Tensile and bending stresses are produced

Types of flat Belt drive

1. open belt drive
2. cross belt drive
3. compound belt drive
4. fast & Loose pulley belt drive
5. stepped pulley drive
6. Jockey pulley drive (open belt drive with idler pulley)
7. Quarter turn belt drive
8. (Right angled belt drive)

Suitable for Medium Centre distance

⇒ 1-6 ⇒ are used for parallel shafts

⇒ 7 ⇒ for Non parallel non intersecting right angled shafts

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open BD ⇒ rotating in same direction

cross BD ⇒ rotating in opposite direction

compound BD ⇒ to get high speed reduction

fast and loose pulley BD ⇒ driven m/c requires intermittent motion

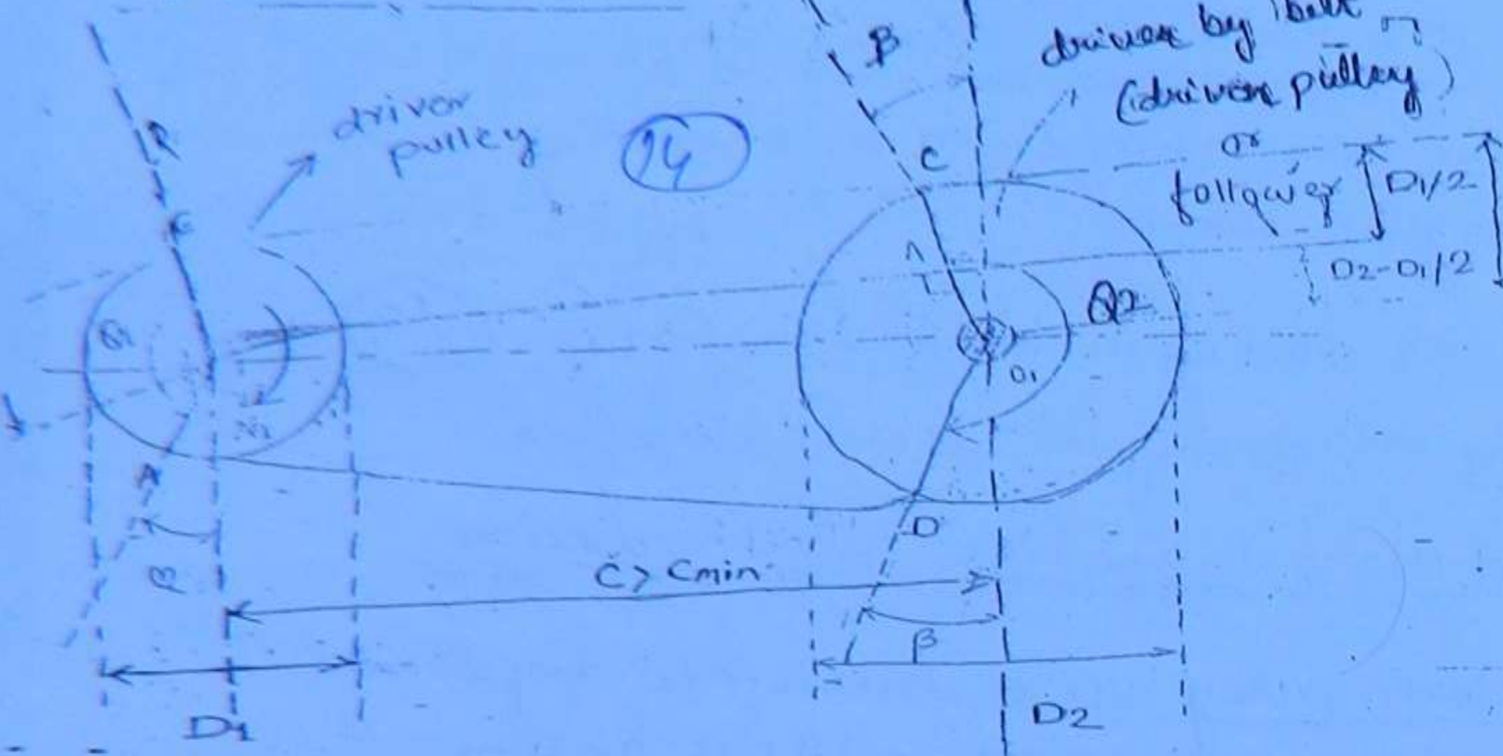
Its function is similar to clutches

stepped pulley drive ⇒ variable speed drive

Jockey pulley drive ⇒ transmit power between two parallel shafts which are at smaller centre distance.

OPEN BELT DRIVE

# OPEN BELT DRIVE




$Q_1 = \pi - 2\beta$

$Q_2 = \pi + 2\beta$

$Q_1 + Q_2 = 2\pi$

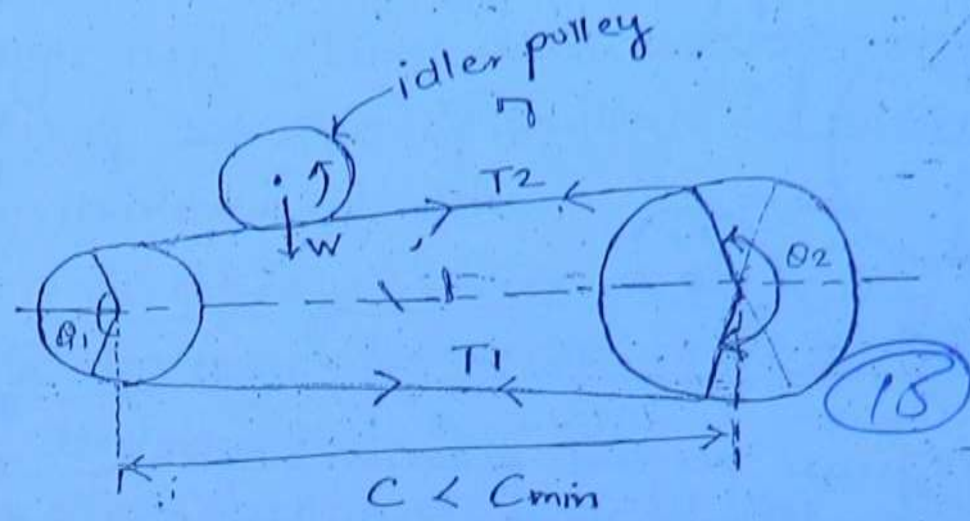
$$\boxed{\beta = \sin^{-1}\left[\frac{D_2 - D_1}{2C}\right] \times \frac{\pi}{180}}$$

$$\sin\beta = \frac{O_1 A}{O O_1} = \frac{D_2 - D_1}{2C}$$

Centre distance decrease, $\beta$ increases

$\theta_1$ decreases, $Q_2$ increases

Slip will occur at driver pulley

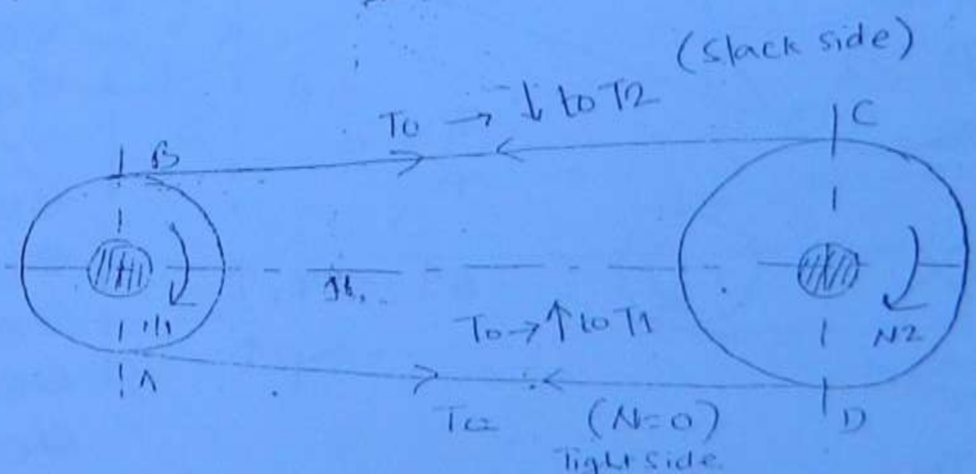



**Length of the Open belt drive**

$L_{OBD}$ = Arc AB + BC + Arc CD + DA

$$\boxed{L_{OBD} = 2C + \frac{\pi}{2}(D_1 + D_2) + \frac{(D_2 - D_1)^2}{4C}} \;**$$

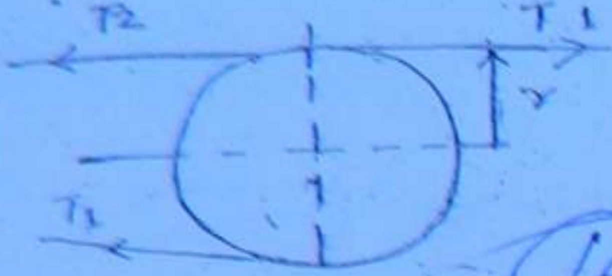
Length choosen is less than $L_{OBD}$ for Initial Tension (by stretching) and power Transmission increases




$N_2 < N_1$

$P = T \cdot \omega$ $\quad$ $\cancel{P \propto \frac{1}{\omega}}$ $\quad$ $T_2 > T_1$

$P = F_t \cdot r\omega$ $\quad$ $T \propto \frac{1}{\omega}$

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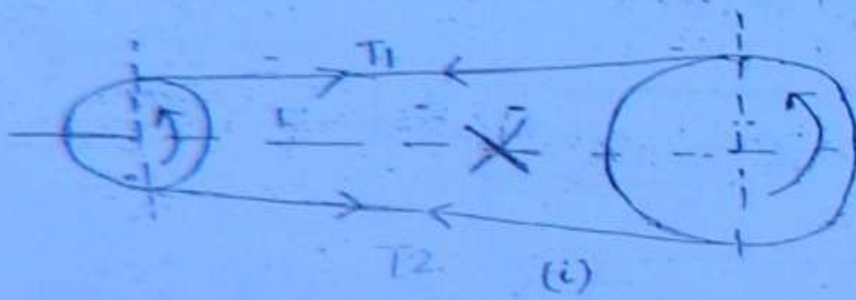
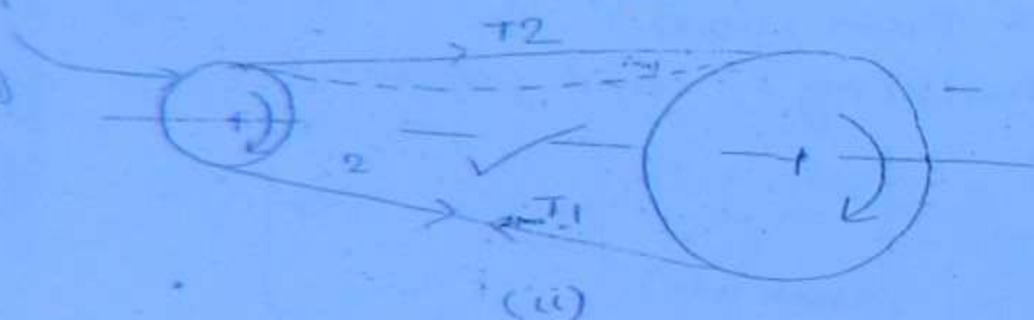
$$P = (T_1 - T_2)v$$

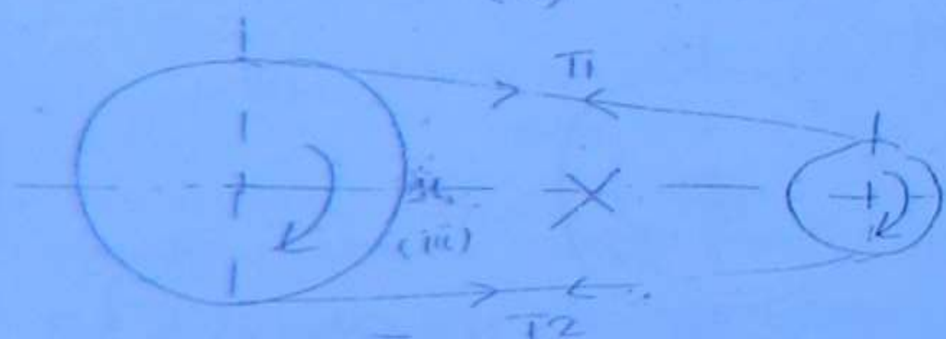
ght. Side ⇒ It is defined as the portion of the elt which is entering the driver pulley leaving the driven pulley




worst



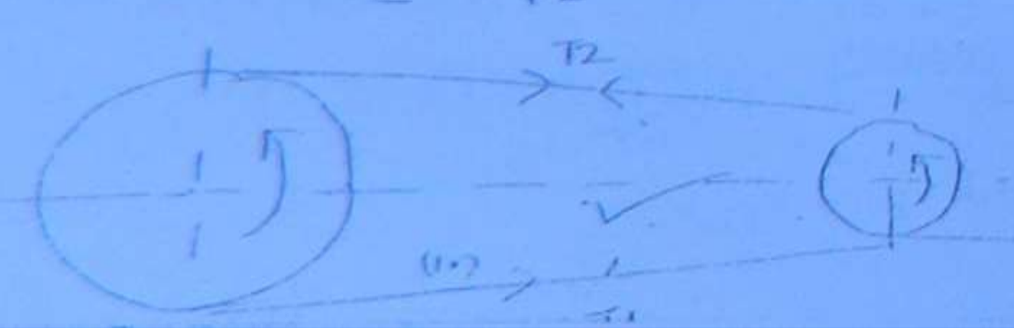

Better




$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$\theta \uparrow \Rightarrow T_1 \uparrow$

$\Rightarrow P \uparrow$



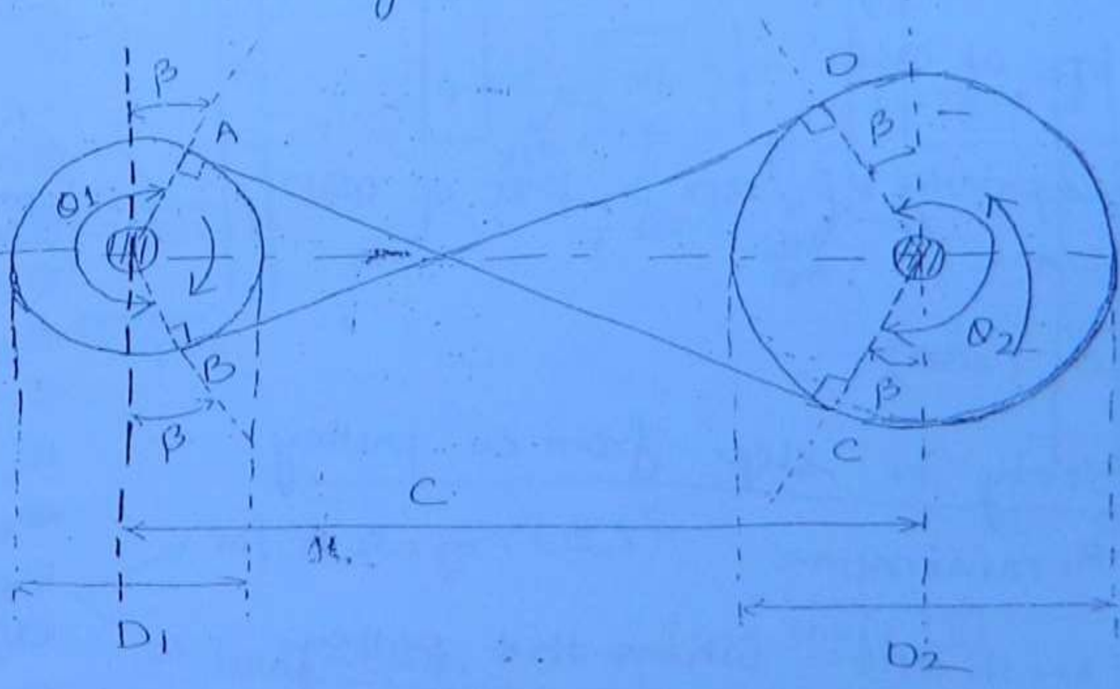

Better

The tight and slack sides are depends upon direction of rotation of pulley as well as position of the driver and driven pulley.

⇒ It is always better to have tight side at the bottom because of the sagging of the belt on the top side (slack side) (?) angle of contact increases at the smaller pulley (as angle of contact increases the $T_1$ increases and hence power will increase.

CROSS BELT DRIVE

To transmit power between parallel shafts which are running in opposite direction.



$\theta_1 = \theta_2 = \pi + 2\beta$

$$\boxed{\beta = \sin^{-1}\left[\frac{D_2 + D_1}{2C}\right] \times \frac{\pi}{180}} \; *$$

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$L_{CBD} = \text{Arc } AB + AC + \text{Arc } CD + DB$

$$\boxed{L_{CBD} = 2C + \frac{\pi}{2}\left(D_1 + D_2\right) + \frac{(D_2 + D_1)^2}{4C}} \; *$$

diff of $L_{OBD}$ and $L_{CBD}$ is

$$\boxed{\Delta L = \frac{D_1 \cdot D_2}{C}}$$

Life of CBD < life of OBD

Power Transmission Capacity (PTC) of CBD > PTC of OBD

$\Rightarrow \theta_1$ CBD > $\theta_1$ (OBD)

Belt is always likely to slip from a pulley where $\mu \cdot \theta$ is minimum

$\Rightarrow$ In a open belt drive when the pulleys are made of same material the belt is likely to slip from smaller pulley because $\mu_1 = \mu_2$ but $\theta_1 < \theta_2$

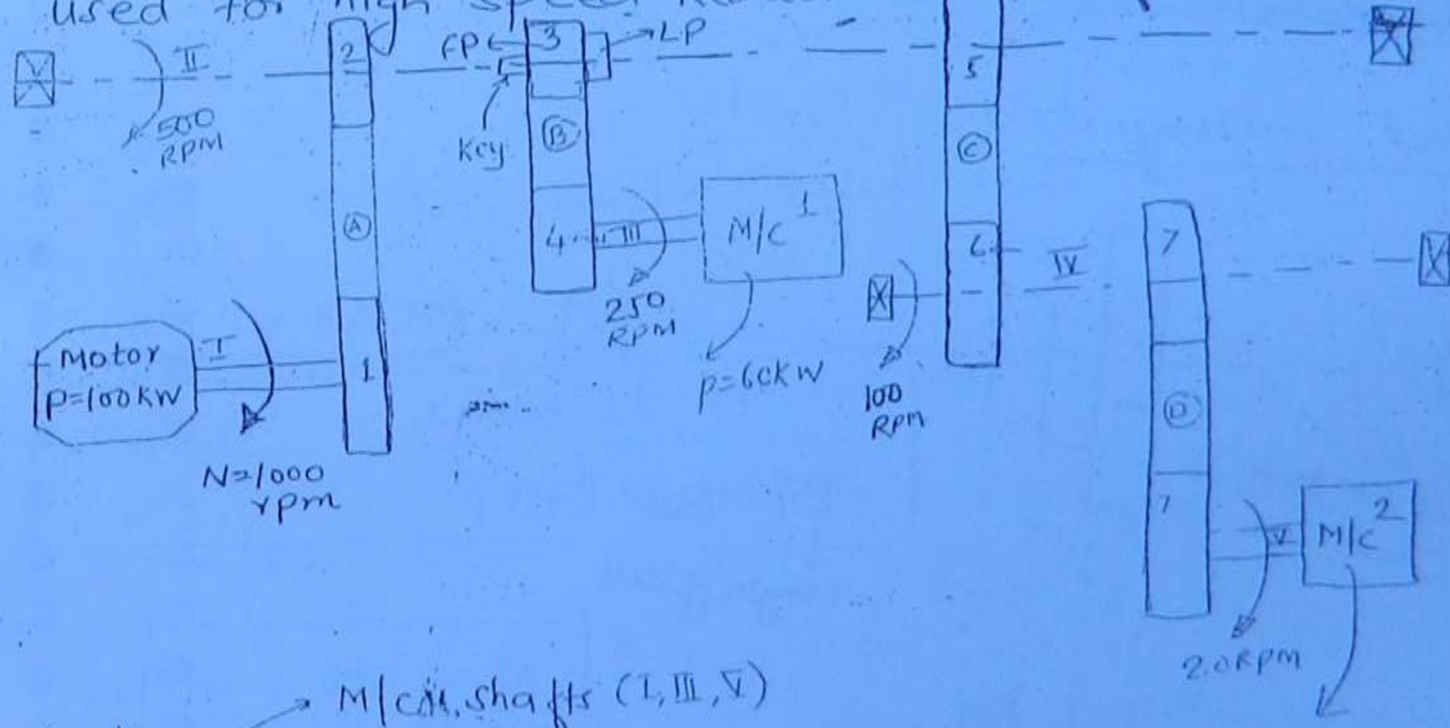
⇒ In a cross belt drive when the pulleys are made up of same material the belt is likely to slip from both the pulley simultaneously because $\mu_1\theta_1 = \mu_2\theta_2$ [$\mu_1 = \mu_2$, $\theta_1 = \theta_2$]

⇒ In a CBD when pulleys are made up of diff material the belt is likely to slip from pulley where $\mu$ is minimum because [$\theta_1 = \theta_2$]

⇒ Always we have to design with respect to pulley where the belt is likely to slip

## COMPOUND BELT DRIVE

used for high speed Reduction




Shafts
- → M/c shafts (I, III, V)
- → Transmission Shafts (II, IV)
  - → Line shaft (II)
  - → Countershaft (IV)

above - fast and Loose pulley also

e shaft It is a intermediate transmission shaft which is used for distributing the power among various machines.

Counter shaft It is a transmission shaft which is used to get the higher speed reduction.

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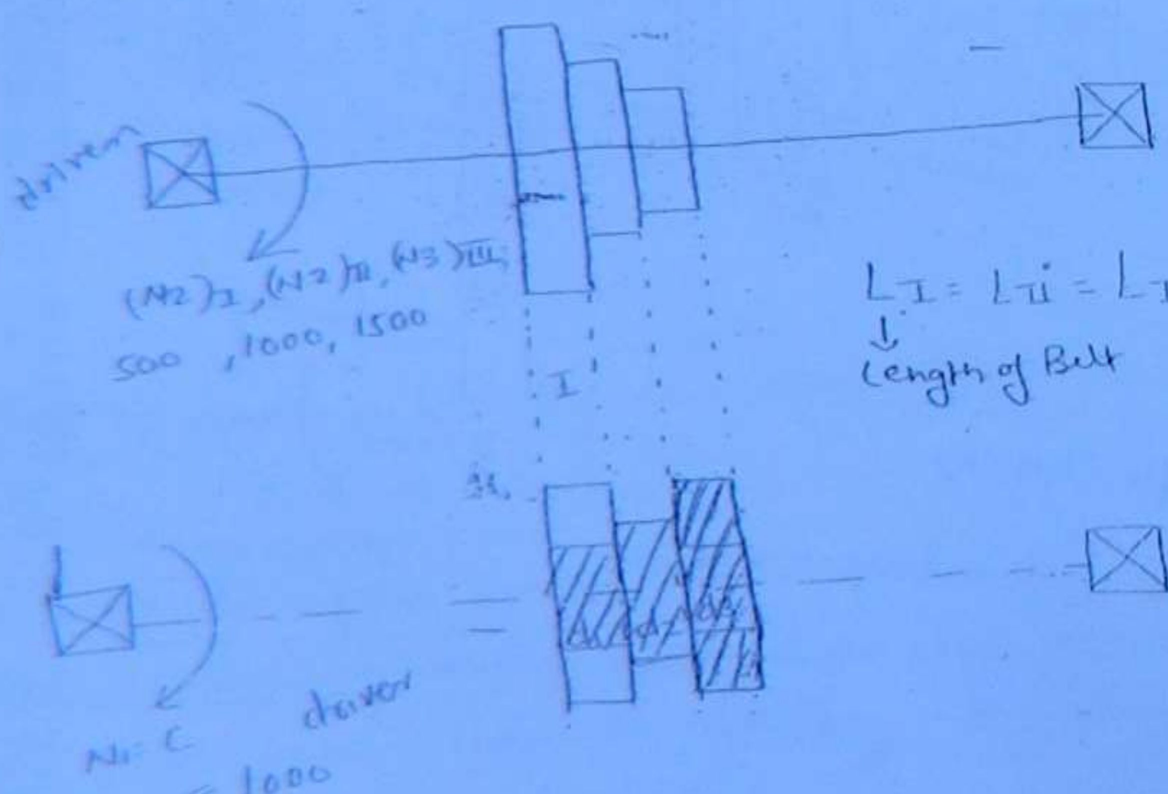
| Fast pulley | Loose pulley |
|---|---|
| Key connection | 1. No Key connection |
| capable of power transmission | 2. incapable of power transmission |

IMP.

## Stepped pulley drive



$\Rightarrow L_I = L_{II} = L_{III}$

$\Rightarrow \frac{(N_2)_I}{N_1} = \frac{d_1}{D_1} \Rightarrow D_1 = ?$

if $d_1$ is known
$D_1$ can be determined

$\frac{(N_2)_{II}}{N_1} = \frac{d_2}{D_2}$ ∴ $d_2 = x \cdot D_2$

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Now $L_I = L_{II}$

$$2C + \frac{\pi}{2}(D_1 + d_1) + \frac{(D_1 - d_1)^2}{4C} = 2C + \frac{\pi}{2}(D_2 + d_2) + \frac{(D_2 - d_2)^2}{4C}$$

$d_2 \rightarrow x D_2$

∴ $D_2$ can be calculated.

## VELOCITY RATIO

$$\boxed{VR = \frac{N_2}{N_1} = \frac{\text{Speed of follower}}{\text{speed of driver}}}$$ *

$$\boxed{VR = \frac{N_2}{N_1} = \frac{D_1}{D_2}}$$ **

equation is valid by neglecting belt thickness effect and slip effect

Let $v_1$ = Linear velocity of driver pulley
$v$ = Linear velocity of belt
$v_2$ = Linear velocity of follower

$V_1 = V = V_2$ [No slip] (22)

$V_1 > V > V_2$ [in presence of slip]

Slip! It is defined as the relative motion between belt and pulley surface, due to insufficient frictional grip. (because of air layer present between pulley and belt surface)

⇒ belt velocity is less than driver pulley velocity but more than driven pulley velocity hence in presence of slip Belt moves somewhat slower than driver pulley but ~~more~~ somewhat faster than the driven pulley.

⇒ In presence of slip speed of the follower decreases, hence velocity ratio of a belt drive and efficiency of belt drive decreases.

as $V_1 = V_2$

$$\frac{\pi D_1 N_1}{60} = \frac{\pi D_2 N_2}{60}$$

$$\boxed{\frac{D_1}{D_2} = \frac{N_2}{N_1}}$$

$$\boxed{\frac{N_2}{N_1} = \frac{D_1 + t}{D_2 + t}}$$

by neglecting effect of slip

## In presence of slip

$$V = V_1 - V_1 \cdot \frac{S_1}{100}$$

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$$V = V_1\left[1 - \frac{S_1}{100}\right] \rightarrow (i)$$ → $S_1$ = percentage of slip between driver pulley and belt.

$$V_2 = V - V\frac{S_2}{100}$$

$S_2$ = % of slip between driven pulley and belt

$$V_2 = V\left[1 - \frac{S_2}{100}\right] \rightarrow (ii)$$

Subst eq (i) in eq (ii) we get

$$V_2 = V_1\left[1 - \frac{S_1}{100}\right]\left[1 - \frac{S_2}{100}\right]$$

$$\frac{\pi(D_2+t)N_2}{60} = \frac{\pi(D_1+t)N_1}{60}\left[1 - \left(\frac{S_1+S_2}{100}\right) + \cancel{\frac{S_1 S_2}{10^4}}\right]$$

neglected

$$\boxed{\frac{N_2}{N_1} = \left(\frac{D_1+t}{D_2+t}\right)\left[1 - \frac{S}{100}\right]}$$ **

S = percentage of total slip of belt drive

$$\boxed{S = S_1 + S_2}$$

## VELOCITY RATIO IN A COMPOUND BELT DRIVE

$$\boxed{VR = \frac{N_n}{N_1} = \left[\frac{(D_1+t)(D_3+t)\cdots\cdots(D_{n-1}+t)}{(D_2+t)(D_4+t)\cdots\cdots(D_n+t)}\right]\left[1-\frac{S}{100}\right]} \; *$$

$$\boxed{S = S_1 + S_2 + \cdots\cdots + S_n} \; *$$

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?: 3 belt drives

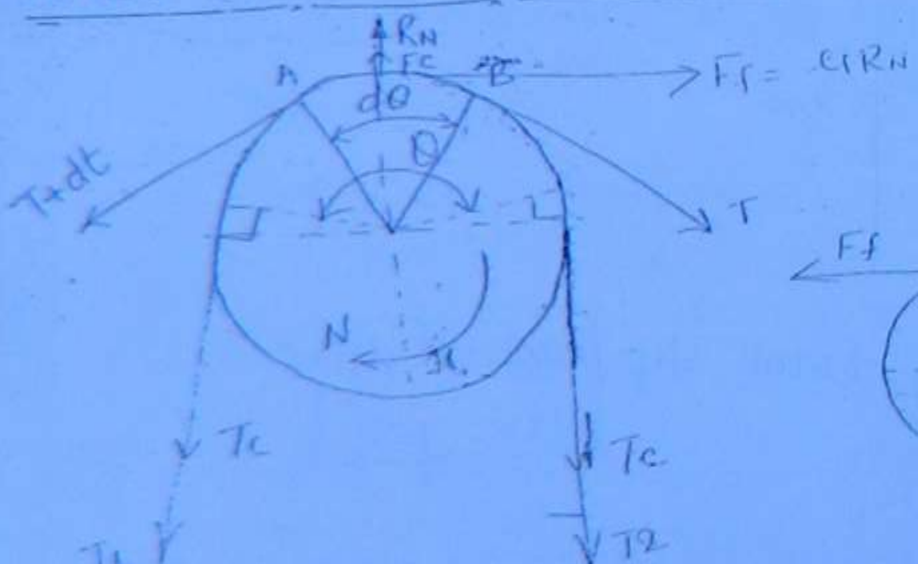
$S = ?$

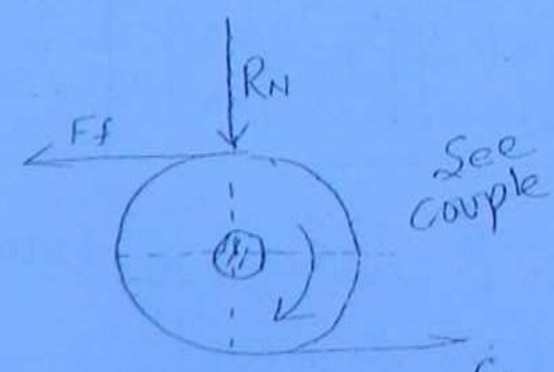
If slip at each belt drive = 2%.

$$S = \underbrace{(S_1 + S_2)}_{2\%} + \underbrace{(S_3 + S_4)}_{2\%} + \underbrace{(S_5 + S_6)}_{2\%}$$

$S = 6\%$

## RATIO OF BELT TENSIONS



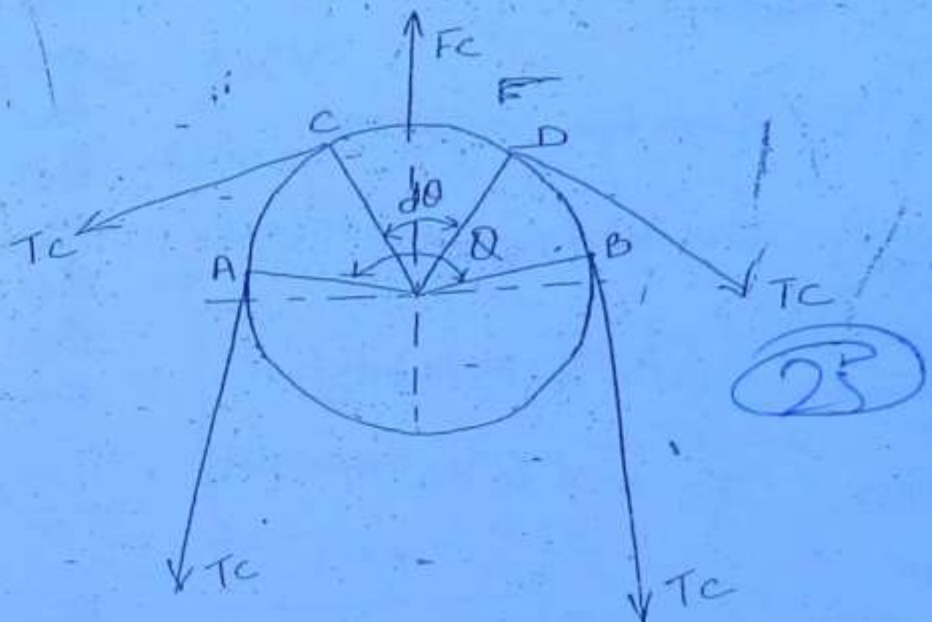

pulley FBD

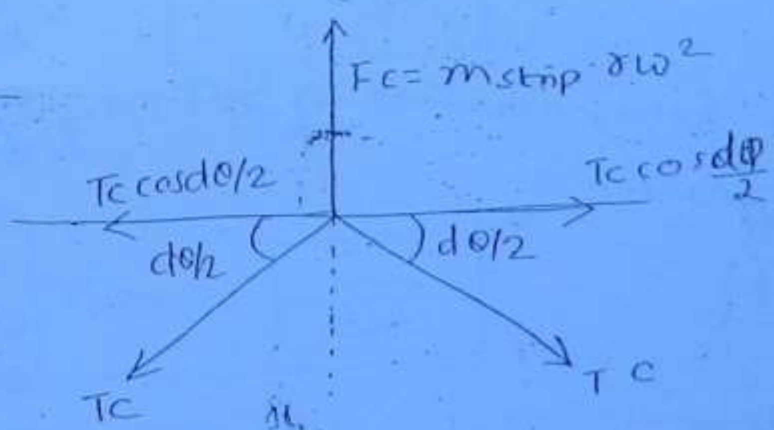
$T_c$ = centrifugal tension

**Centrifugal Tension**: additional tension induced in the belt in presence of centrifugal force

$T_t$ = Total tension on Tight side = $T_1 + T_c$

$T_s$ = Total tension on slack side = $T_2 + T_c$




$$\Sigma V = 0$$

$$F_c - T_c \sin\frac{d\theta}{2} - T_c \sin\frac{d\theta}{2} = 0$$

$$m_{strip}\, r\omega^2 - T_c \cdot \frac{d\theta}{2} - T_c \frac{d\theta}{2} = 0$$

$$m_{strip}\, r\omega^2 - T_c\, d\theta = 0$$

Let $m$ = mass of the belt per unit length in kg/m

$$m_{strip} = m \cdot r \, d\theta$$

$$m \cdot r \, d\theta \cdot r\omega^2 - T_c \cdot d\theta = 0$$

$$m r^2 \omega^2 \cancel{d\theta} = T_c \cancel{d\theta}$$

$$\boxed{mv^2 = T_c}$$

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$V < 8$ m/s $\Rightarrow$ Effect of $T_c$ can be neglected

$V \geq 8$ m/s $\Rightarrow$ Effect of $T_c$ should be considered

$m = \rho \cdot V$

$m = 1000 \times A \times L$

$\rho_{leather} = (950 \text{ to } 1050)$ kg/m$^3$

$\sigma_{per} = \frac{S_{yt}}{N} = 2$ to $2.5$ MPa

$$m = 1000 \times \frac{\overset{mm}{b}}{1000} \times \frac{\overset{mm}{t}}{1000} \times 1 \text{ m}$$

$T_{max}$ = maximum Tensile force, material of belt can withstand without failure

for safe design of belt

$$[(\sigma_t)_{max}]_{ind} \leq (\sigma_t)_{per}$$

$$\frac{T_1}{A} \leq (\sigma_t)_{per}$$

or $\frac{T_1 \text{ or } T_t}{A} \leq (\sigma_t)_{per}$

## Condition for Maximum Power Transmission

$$P = (T_1 - T_2)V$$

$P \propto V$ as $V \uparrow \Rightarrow P \uparrow$

but as $V \uparrow \Rightarrow T_c \uparrow \Rightarrow P \downarrow$

Now $P = T_1 K' V$

$$P = (T_{max} - T_c) K' V = T_{max} K' V - K' m V^3$$

$$\frac{dP}{dV} = 0 \Rightarrow T_{max} \cdot K' - 3K' m V^2 = 0$$

$$K' [T_{max} - 3T_c] = 0$$

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$$K' \neq 0 \Rightarrow T_{max} - 3T_c = 0$$

$$\Rightarrow T_{max} = 3T_c$$

$$\boxed{T_c = \frac{T_{max}}{3}} \; *$$

$$m V_{max}^2 = \frac{T_{max}}{3}$$

$$\boxed{V_{max} = \sqrt{\frac{T_{max}}{3m}}} \; *$$

**$P_{max}$:**

(1) $T_{max} = \sigma_{per} \cdot b \cdot t = \underline{\ ?\ } N$

(2) $T_c = \frac{T_{max}}{3} = \underline{\ ?\ } N$

(3) $T_1 = T_{max} - T_c = 2T_c$

$\frac{T_1}{T_2} = e^{(\mu\theta)_{min}} = K$ ⑤ $m = \rho \cdot \frac{b}{1000} \times \frac{t}{1000} \times 1m$

$T_2 = \frac{T_1}{K}$ ⑥ $V_{max} = \sqrt{\frac{T_{max}}{3m}}$

$P_{max} = (T_1 - T_2)\, V_{max}$

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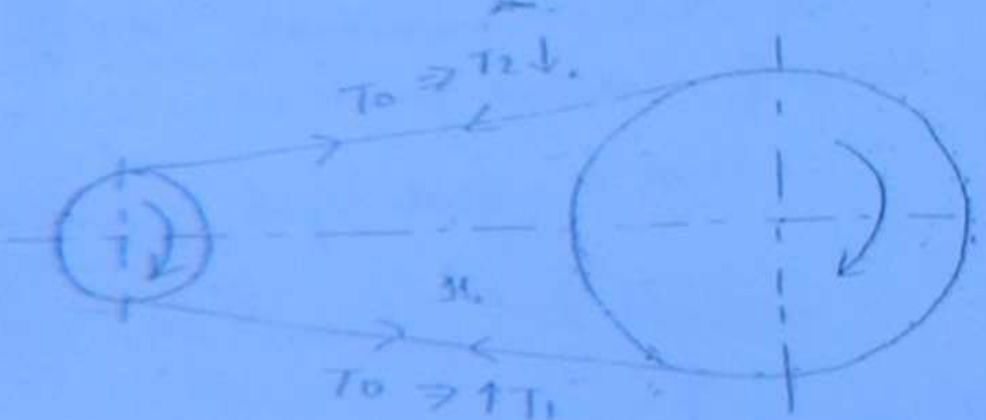
## Expression for Initial Tension ($T_o$)

Initial Tension is the tension induced in the belt when it is in the stationary condition. It is provided in the belt by taking a length of belt by taking a length of belt less than the actual required length.

$\left\{ \begin{array}{l} \text{as } L\downarrow \Rightarrow T_o \uparrow \Rightarrow F_f \uparrow \Rightarrow \frac{T_1}{T_2}\uparrow \\ T_1 \uparrow \text{ or } T_2 \downarrow \text{ and } P \uparrow \end{array} \right\}$




Increase in length of belt on tight side = decrease in length of belt on slack side

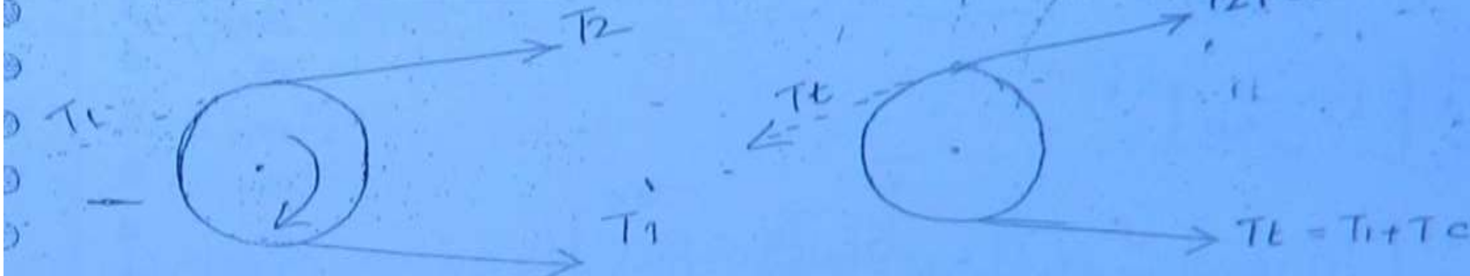
$T_1$ or $T_t \leq A \cdot (\sigma_t)_{per}$

$T_1$ or $T_t \leq (b \cdot t \cdot (\sigma_t)_{per}) \rightarrow T_{max}$

$\therefore \boxed{T_{max} = b \cdot t \cdot (\sigma_t)_{per}}$ **

Power Transmission capacity (PTC):- 29

$$\boxed{PTC = (T_1 - T_2)\,V} \longrightarrow (i)$$




$$P = (T_t - T_s)\,V$$

$$P = [(T_1 + T_c) - (T_2 + T_c)]\,V$$

$$\boxed{P = (T_1 - T_2)\,V} \longrightarrow (ii)$$

$T_1$ or $T_t \leq T_{max}$

$\Rightarrow T_c = 0 \Rightarrow T_1 = T_{max}$

$\Rightarrow T_c \neq 0 \Rightarrow T_t = T_{max}$

$\Rightarrow T_1 + T_c = T_{max}$

$\Rightarrow \boxed{T_1 = T_{max} - T_c}$

## Effect of Tc on PTC

$T_c = 0 \Rightarrow T_1 = T_{max}$

$T_c \neq 0 \Rightarrow T_1 = T_{max} - T_c$

$$P = (T_1 - T_2)V$$

$$P = T_1\left[1 - \frac{T_2}{T_1}\right]V$$

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$$P = T_1\left[1 - \frac{1}{T_1/T_2}\right]V$$

$$P = T_1\left[1 - \frac{1}{e^{\mu\theta}}\right]V$$

$$P = T_1\left[1 - \frac{1}{K}\right]V \qquad \left[\because K = \frac{T_1}{T_2} = e^{\mu\theta}\right]$$

$$\boxed{P = T_1 \cdot K' V} \qquad K' = \left(1 - \frac{1}{K}\right)$$

when $T_c = 0$

$$\boxed{P = T_{max} \cdot K' V} \longrightarrow (i)$$

$$T_c \neq 0 \Rightarrow \boxed{P = (T_{max} - T_c) K' V} \longrightarrow (ii)$$

from the above two eqn we can conclude that presence of centrifugal tension power transmission capacity of belt drive decreases and hence with respect PTC of a belt drive centrifugal tension is harmful

$\alpha$ = coefficient of change in length belt / unit force

$$(T_1 - T_0)\alpha = (T_0 - T_2)\alpha$$

$$\boxed{T_0 = \frac{T_1 + T_2}{2}} \; *$$

Considering centrifugal Tension

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$$\boxed{T_0 = \frac{T_1 + T_2 + 2T_c}{2}}$$

Design procedure used in flat Belts

1) $VR = \frac{N_2}{N_1}$ -

2) Determination of Dia. of pulleys [either $D_1$ or $D_2$] -

$$\frac{N_2}{N_1} = \left(\frac{D_1 + t}{D_2 + t}\right)\left[1 - \frac{S}{100}\right]$$

$D_1$ or $D_2$ can be determined

3) Thickness of belt

$$(\sigma_b)_{max} \leq (\sigma_t)_{per}$$

$$\frac{Et}{D_1} \leq (\sigma_t)_{per}$$

$$t \leq \_\_\_ \text{ mm}$$

4) belt velocity ($v$)

(a) In the absence of slip

$$v = v_1 \text{ or } v_2 = \frac{\pi(D_1 + t)N_1}{60 \times 1000} \text{ or } \frac{\pi(D_2 + t)N_2}{60 \times 1000} = ? \text{ m/s}$$

i) In presence of slip

$v = v_1\left[1 - \frac{s_1}{100}\right]$ or $v_2 = v\left[1 - \frac{s_2}{100}\right]$

$v = \underline{\ ?\ }$ m/s

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$T_{max} = \underset{\text{mpa}}{\sigma_{per}} \cdot \underset{\text{mm}}{b} \cdot \underset{\text{mm}}{t} = \underline{\ ?\ }\ N = b\ \text{in}\ N$

$m = \underset{\text{kg/m}^3}{\rho} \cdot \frac{b}{1000} \cdot \frac{t}{1000} \cdot 1m = \underline{\quad}\ b\ \text{in kg/m}$

$T_c = mv^2 = \underline{\quad}\ b\ \text{in}\ N$

$\therefore T_1 = T_{max} - T_c = \underline{\quad}\ b\ \text{in}\ N$

$T_2 - ?$

$$\frac{T_1}{T_2} = e^{(\mu\theta)_{min}} \quad \text{rad}$$

OBD, $\theta_1 = \pi - 2\left[\sin^{-1}\left(\frac{D_2 - D_1}{2C}\right) \times \frac{\pi}{180}\right]$

$\theta_1 = \underline{\quad}$ radian

$\theta_2 = 2\pi - \theta_1 = \underline{\quad}$ rad

$\mu_1\theta_1 = \underline{\quad}$

$\mu_2\theta_2 = \underline{\quad}$

$(\mu\theta)_{min} = \text{min of}\left[\mu_1\theta_1\ \&\ \mu_2\theta_2\right]$

In CBD

$$\theta_1 = \theta_2 = \pi + 2\left[\sin^{-1}\left(\frac{D_2 + D_1}{2C}\right) \times \frac{\pi}{180}\right]$$

$e_{1}\theta_1 =$ —

$e_{2}\theta_2 =$ —

$(e_1\theta)_{min}$ = $\theta_1$ or $\theta_2 \times$ [min of $e_1$ and $e_2$]

$$\frac{T_1}{T_2} = e^{(e_1\theta)_{min}} = K$$

B3

$$\therefore T_2 = \frac{T_1}{K} = \text{—} \; b \text{ in } N$$

(10) b -

$P = (T_1 - T_2) v$

in watt $P = (\text{—} \; b) v$

$\therefore b = \underline{99.1}$ mm , can be calculated

$b = 100$ mm

(11)

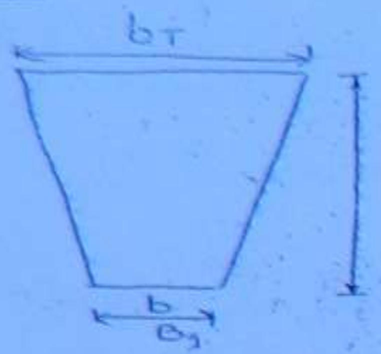
$$L_{OBD} = 2C + \frac{\pi}{2}(D_1 + D_2) + \frac{(D_2 - D_1)^2}{4C} = \text{—} \, mm$$

$$L_{CBD} = 2C + \frac{\pi}{2}(D_1 + D_2) + \frac{(D_2 + D_1)^2}{4C} = \text{—} \, mm$$

SD2

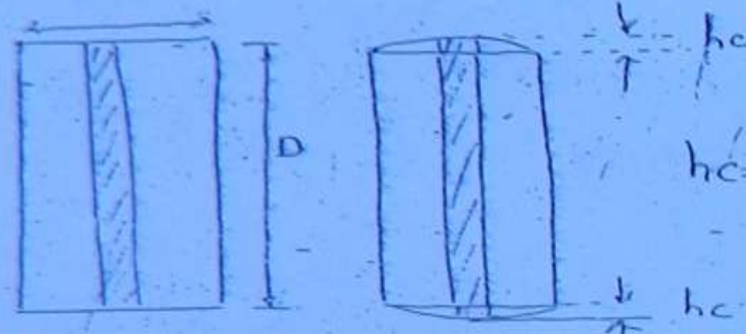
: A Tra

## V-BELTS



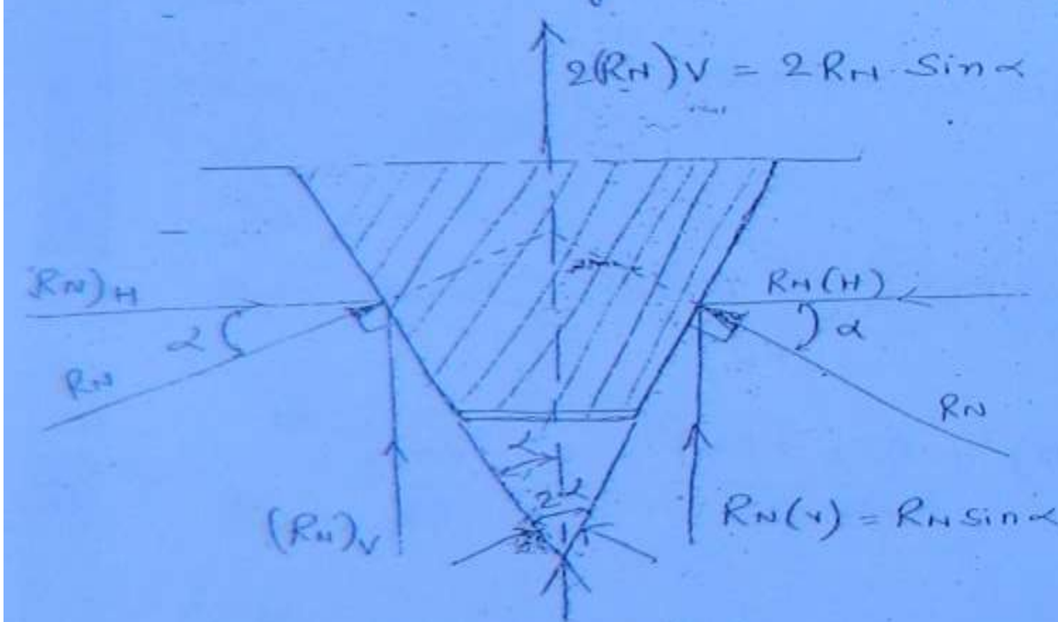

B4

## ROWNING



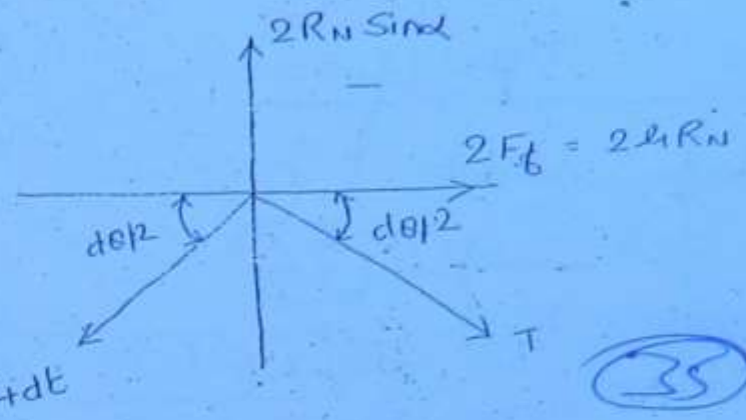

hc= crownhead or crown height

flat pulley          crownpulley




$2\alpha$ = groove angle
$= 36° \text{ to } 42°$
$= 40°$

$\alpha$ = Semi groove angle

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$\Sigma H = 0$, $\Sigma V = 0$

We get, $$\boxed{\frac{T_1}{T_2} = e^{\frac{\mu\theta}{\sin\alpha}}}$$ **

$\alpha \simeq 20^\circ$ | $\sin 20^\circ < 1 \therefore \frac{\mu\theta}{\sin\alpha} > \mu\theta$

$\Rightarrow \left(\frac{T_1}{T_2}\right)_{V Belt} > \left(\frac{T_1}{T_2}\right)_{flat\ belt}$

$\Rightarrow PTC_{V Belt} > PTC_{flat\ belt}$

$$\boxed{P_{Design} = P_T \times K_a}$$

$K_a$ = overload / service factor

$P_T$ = Power to be Transmitted

$\Rightarrow$ In case of Multiple V- belts even if a single belt gets damaged entire set of the V- belts has to be replaced by a complete new set of V- belts to ensure uniform tension in all the belts.

No of 'V' belts = (n) $$\boxed{n = \frac{P_{Total}}{P_{each}}}$$

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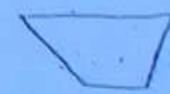
| Parameter | FLAT BELTS | V-BELTS |
|---|---|---|
| Centre distance | Medium | Short |
| Cross section |  |  |
| $T_1/T_2$ | less $T_1/T_2 = e^{\mu\theta}$ | More $T_1/T_2 = e^{\mu\theta/\sin\alpha}$ |
| Slip | occurs | rare |
| m | less | more |
| Cost | less | more |
| pulley | flat pulley | grooved pulley |
| Idler pulley | | |
| No. of belt | one or Two | Multiple v belts |
| Noise | More, because of Joint | Less, (endless belt) No Joint |

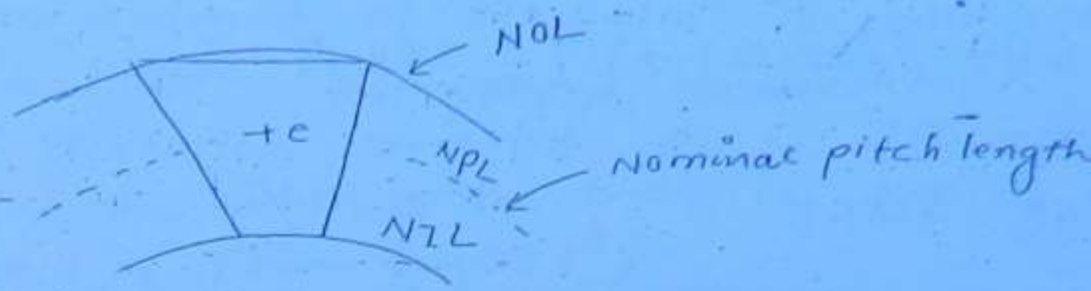
V Belt designation

B- 3638 - Gr.52

3 ⇒ Type of v belt

638 ⇒ nominal inside length (NIL)

. 52 ⇒ Grade number (oversized belt)




$$\boxed{NPL = NIL + K}$$

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(2) B-3638 - Gr-52 ⟶ oversize

(3) B- 3638 - Gr.50 ⟶ Standard size

(4) B- 3638 - G.46 ⟶ undersize,

Manufacturing Length (ML) = NPL ⇒ std. size belt ⇒ Gr50

ML > NPL ⇒ oversize belt ⇒ Gr >50

ML < NPL ⇒ undersize belt ⇒ Gr <50

⇒ 1 Grade Number is deviation from standard size (50) is equal to 2.5 mm variation

$$\boxed{ML = NPL \pm \left[\text{Difference in Grade No.} \times 2.5\right]}$$

$$\boxed{ML = (NIL + K) \pm (\text{Diff in Grade No} \times 2.5)}$$

Type of V belt

| | dimensions | PTC | Cost | K |
|---|---|---|---|---|
| A | | | | 36 |
| B | | | | 43 |
| C | Increase | | | 56 |
| D | | | | 79 |
| | | | | 92 |

$\frac{ML}{A(2)} = (3638 + 43) + [(52-50)\times 2.5] = 3681+5 = 3686$

$\frac{ML}{m3} = 3638 + 43 = 3681$

$\frac{ML}{4} = (3638+43) - ((50-46)\times 2.5) = 3671$

Calculation of No. of belts

B8

n = no. of v belts

$$\boxed{n = \frac{P_{Total} \times Ka}{P_{each}}}$$

$P_{each} = (T_1 - T_2) V$

$$\boxed{n = \frac{P_{Total} \times Ka}{P_{each} \times kb \times kc}}$$

kb = arc of contact factor

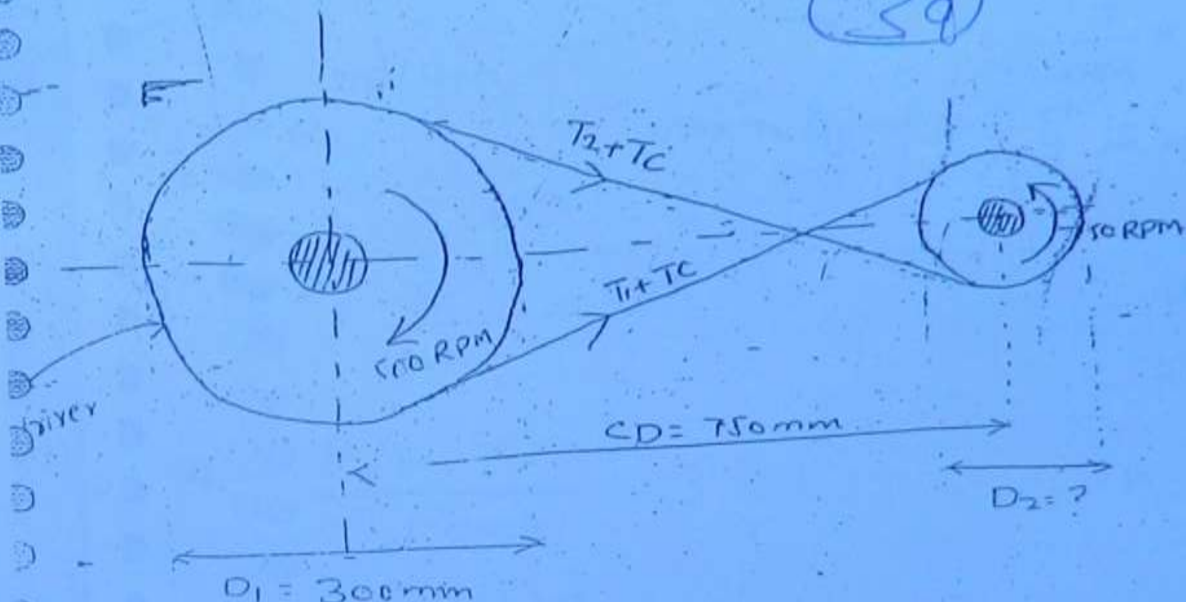
kc = length correction factor

> Peach belt is Calculated by taken $\theta = 180^\circ$ or $\pi^c$

which has a density of 970 kg/m$^3$, The allowable stress is 2MPa Two pulleys rotates in opposite direction and the CD is 750 mm determine the width of the belt by taking coefficient of friction is 0.3 ?

Soln

(39)




① $VR = \frac{N2}{N1} = \frac{750}{500} = 1.5$

(ii) $D2 = ?$

$$\frac{N2}{N1} = \left(\frac{D_1+t}{D_2+t}\right)\left[1 - \frac{S}{100}\right]$$

$$1.5 = \left[\frac{300+4.75}{D2+4.75}\right]\left[1-0\right]$$

$\therefore D2 = 198.4 = 200$ mm

③ $v = v1$ or $v2$

$$v = \frac{\pi(D_1+t)N_1}{60\times1000} = \frac{\pi(300+4.75)\times500}{60\times1000}$$

$$v = 7.98 \text{ m/s}$$

$T_{max} = 6_{per} \times b \times t$

$= 2 \times b \times 4.75$

$\therefore T_{max} = 9.5\, b$ in N

$m = \rho \times \frac{b}{1000} \times \frac{t}{1000} \times 1\, m$ 40

$= 970 \times \frac{4.75}{1000} \times \frac{t}{1000} \times 1$

$m = 4.607 \times 10^{-3}\, b$ in Kg/m

$T_c = m v^2$

$= 0.293\, b$ in N

$T_1 = T_{max} - T_c$

$= 9.207\, b$ in N

$\frac{T_1}{T_2} = e^{(\mu\theta)_{min}}$

$\theta_1 = \theta_2 = \pi + 2\left[\sin^{-1}\left(\frac{D_2 + D_1}{2C}\right) \times \frac{\pi}{180}\right]$

$\therefore \theta_1 = \theta_2 = \cancel{0.338}\ 3.82$ radian

$\frac{T_1}{T_2} = 3.15$

$T_2 = \frac{T_1}{3.15} = 2.933\, b$ in N

$P = (T_1 - T_2) v$

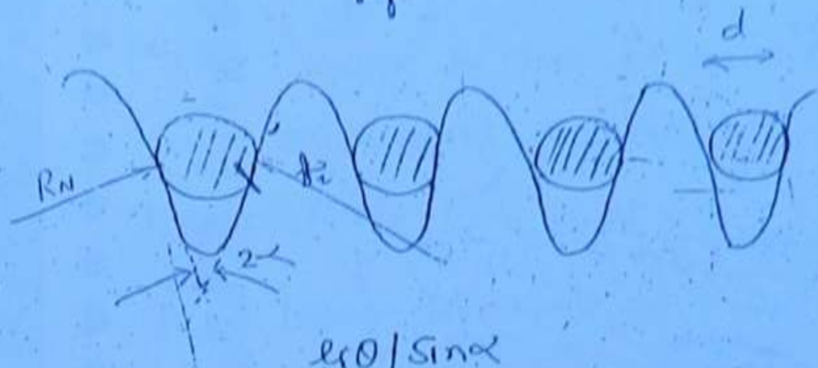
$3.75 \times 10^3 = (9.207 b - 2.933 b)\, 7.98$

$\therefore b = 74.71$ mm $\simeq 75$ mm

$L_{CBD} = 2C + \left(\frac{\pi}{2}(D_1 + D_2)\right] + \frac{(D_2 + D_1)^2}{4C}$

$=$

# DESIGN OF FIBRE ROPE




4

$\Rightarrow \frac{T_1}{T_2} = e^{\mu\theta/\sin\alpha}$ , $\Rightarrow T_{max} = \sigma_{per} \times \frac{\pi}{4} d^2$

$\Rightarrow m = \rho \times \frac{\frac{\pi}{4} d^2}{10^6} \times 1\,m$ (d → mm)

$\Rightarrow n = \frac{P_{Total} \times K_a}{P_{each}}$

$\Rightarrow P_{each} = (T_1 - T_2)\,V$

$\Rightarrow T_1 = T_{max} - T_C$

$\Rightarrow T_C = mv^2$

$\Rightarrow T_2 = \frac{T_1}{e^{\mu\theta/\sin\alpha}}$

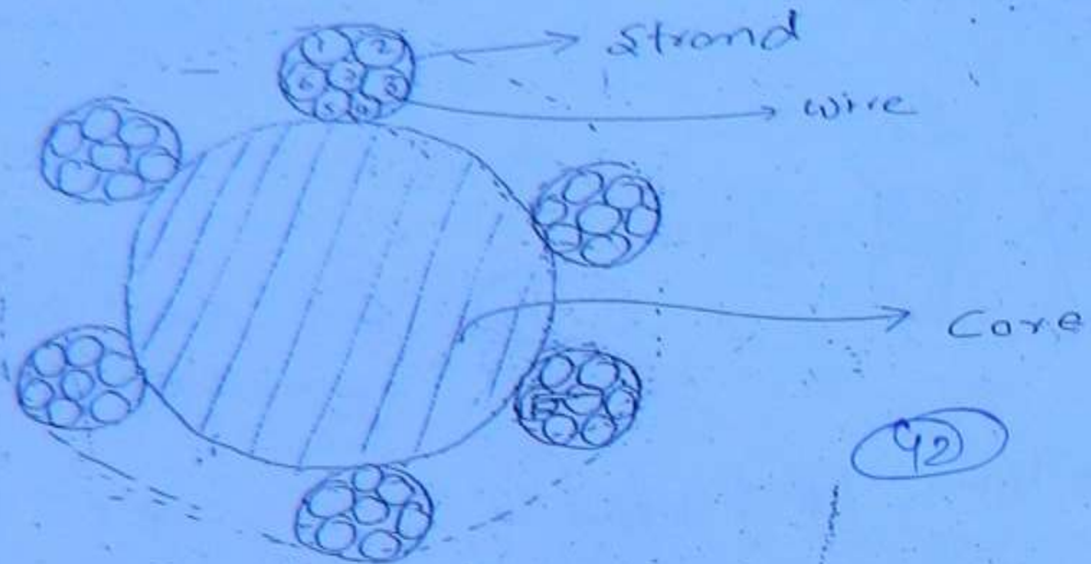
## WIRE ROPES

used in hoisting applications

**Designation**

$\underbrace{6\times7}_{}$ , $6\times19$

- 6: No of strands
- 7: No of wire in each strands

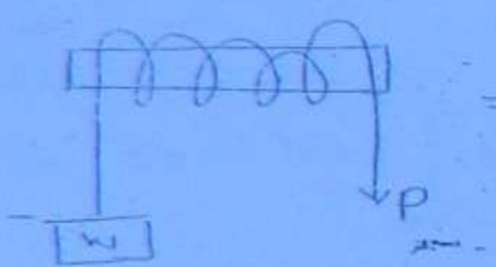



(42)

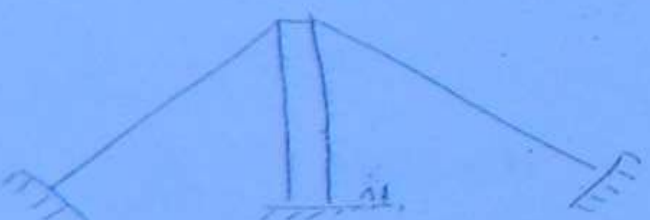
6×7

7 = Small number of large diameter wire.
→ (high stiff)

19 = Large no. of small diameter wire.
→ (more flexible).




⇒ (6×19) flexible



⇒ (6×7) stiffness.

(pole)

A Transmission shaft rotating at 500 RPM
es a milling machine which requires 3.75
at 750 RPM a 300 mm diameter CI pulley
mounted on the Transmission shaft an
(mm)
al design purposes, a belt of 4.75 thickness

IES-07

$$P_D = P_T \times K_a \times K_{FL}$$

$P_T$ = Power to be transmitted

$K_a$ = overload factor

$K_{FL}$ = friction loss

Take less thickness (as bending stresses will be less)

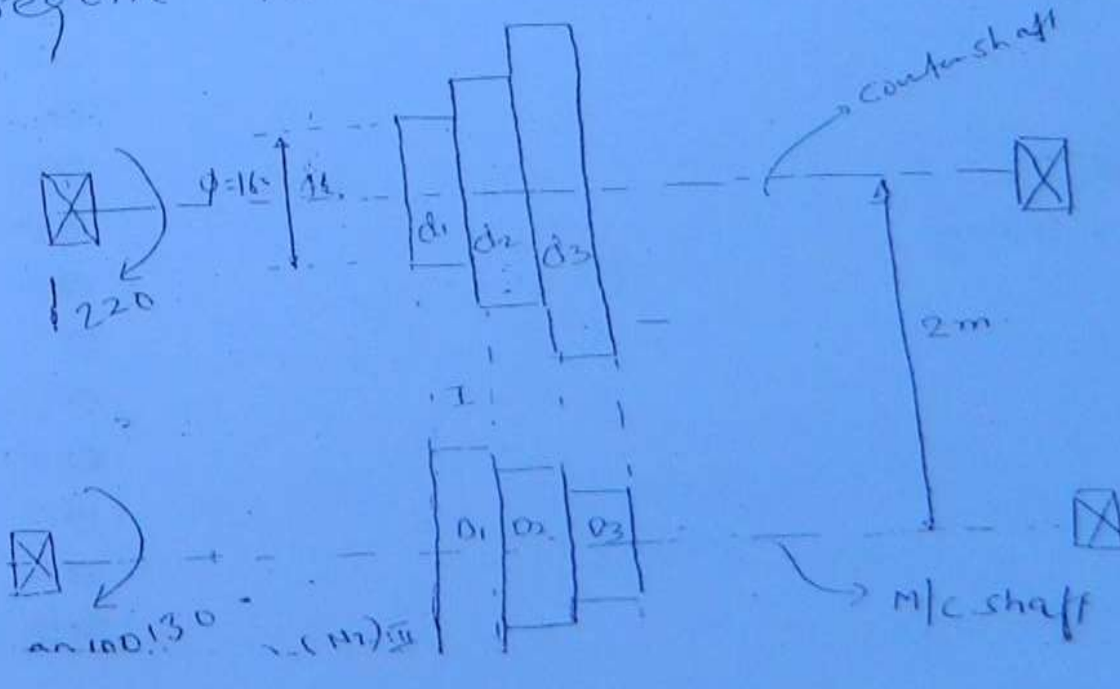
$\hookrightarrow \sigma_b = \frac{Et}{D}$

IES-2007, $\left\{ \begin{array}{l} b = 323.3 \simeq 325\ mm \\ L = 8.83\ m \end{array} \right\}$ Q3

IES-08

Q:2: design a set of stepped pulley to drive a machine from a countershaft that runs at 220 RPM, The CD between 2 sets of pulleys is 2m the diameter of the smallest step of the countershaft is 160 mm. The m/c is to run at 80, 100, and 130 RPM and should be able to rotate in either direction find the length of the belt require for both the cases?

Soln



step

$$VR = \frac{(N_2)_I}{N_1} = \frac{d_1}{D_1}$$

$$\frac{80}{220} = \frac{160}{D_1} \quad \therefore D_1 = 440\,mm$$

$$L_{OBD} = L_I = L_{II} = L_{III}$$

$$= 2C + \frac{\pi}{2}(D_1 + d_1) + \frac{(D_1 - d_1)^2}{4C}$$

$$L_{OBD} = 4.952\,m$$

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nd step

$$VR = \frac{(N_2)_{II}}{N_1} = \frac{d_2}{D_2}$$

$$VR = \frac{100}{220} = \frac{d_2}{D_2}$$

$$\therefore D_2 = 2.2 d_2$$

$$L_{II} = 2C + \frac{\pi}{2}(D_2 + d_2) + \frac{(D_2 - d_1)^2}{4C} = 4.952\,m$$

$$\therefore d_2 = 189\,mm$$

$$D_2 = 417\,mm$$

3rd step

$$d_3 = 380\,mm$$

$$D_3 = 224.3\,mm$$

CREEP

when the belt is in running condition the tensions in the belt changes from $T_1$ to $T_2$ and $T_2$ to $T_1$ and so on, due to this varying tensions in the belt, the belt is subjected to uneven extensions and contractions hence length ~~recieved b y the~~ belt recieved and delivered by a pulley are unequal because of ~~leng~~ difference in length being rcaived and delivered by a pulley relative motion takes place between belt and pulley surfaces. this relative motion or length being recieved and delired by a pulley is called as a creep. Due to creep speed of the follower decreases (i.e, velocity ratio and power transmission capacity reduces). The effect of creep is similar to the effect of ~~creep~~ slip. Hence the combined effect generally called as a slip effect.

generally it is $3\frac{1}{2}$ to 4%

7

## ② DESIGN OF SPUR GEARS

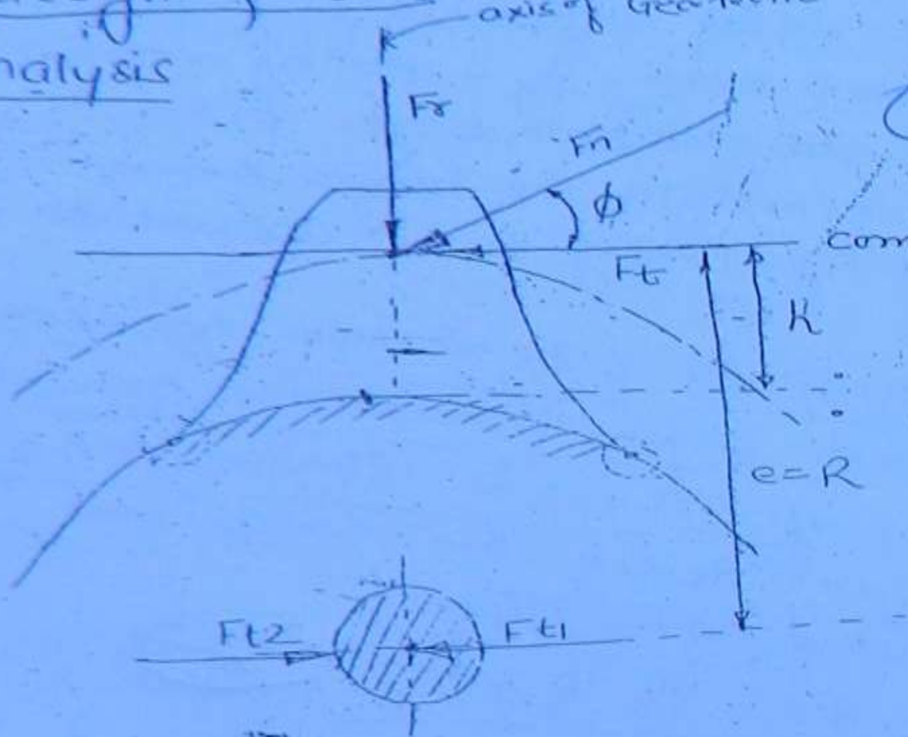
→ To determine the dimensions of a Gear Tooth & Gear (i.e, a, d, $P_c$, $P_d$, b, ~~beretash~~ backlash, D) clearence

→ To determine the above dimensions module is required.

→ To determine 'm' ⇒ Lewis or Beam strength eqn. or design equation.

### FORCE Analysis

(46)




$F_r = F_n \sin\phi \Rightarrow F_r = F_t \tan\phi$

$F_t = F_n \cos\phi$

$\Rightarrow F_n = \dfrac{F_t}{\cos\phi}$

w.r.t Gear, $F_r$ is a axial compressive load

$F_t$ is a TSL

$\Rightarrow$ due to $F_r \Rightarrow \sigma_{ac}$

$\Rightarrow$ due to $F_t \Rightarrow \sigma_b$ and $\tau_s$

always $\cos\phi > \sin\phi$ [$\because \phi = 20°, 14\frac{1}{2}°$]

and hence effect of $F_r$ is neglected

∴ $\sigma_{ac}$ is neglected

Gear tooth is designed by considering bending stresses only

Shaft: $F_r$ is TSL

$F_t$ is eccentric TSL

$F_{t1} = F_{t2} = F_t$

$F_t$ and $F_{t2}$ produces Twisting Moment (TM)

$TM = F_t \cdot e = F_t \cdot R$

finally shaft is subjected to

⇒ (i) Twisting moment

⇒ (ii) BM in vertical plane

⇒ (iii) BM in horizontal plane

Design a GD, $P = x$ KW at $y$ RPM ? (default pinion RPM)

(i) $T_1 = \frac{P \times 60}{2\pi N_1} \times 10^6$ —— N-mm

(ii) $T = F_t \cdot R$

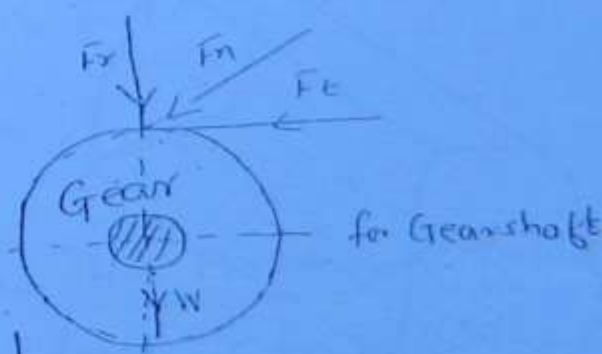
$F_{t_1} = \frac{T}{R} = \frac{2T}{D_1}$

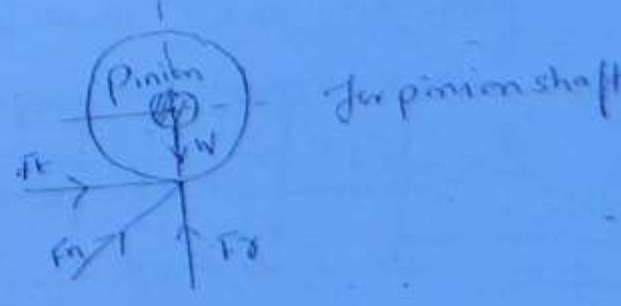
$F_{t2} = \frac{2T_2}{D_2}$

$F_{t1} = F_{t2}$

$\frac{\cancel{2}T_1}{D_1} = \frac{\cancel{2}T_2}{D_2}$

$\boxed{\frac{T_1}{\ } = \frac{D_1}{\ }}$



$$\frac{T_1}{T_2} = \frac{D_1}{D_2} = \frac{Z_1 m_1}{Z_2 m_2}$$ $\left\{ Z_1, Z_2 \right.$ = Teeth on pinion and Gear

$$\boxed{\frac{T_1}{T_2} = \frac{D_1}{D_2} = \frac{Z_1}{Z_2}}$$ Torque on Gear will be more as D is more

i) $F_r = F_t \cdot \tan\phi$

ii) $F_n = \sqrt{F_t^2 + F_r^2}$

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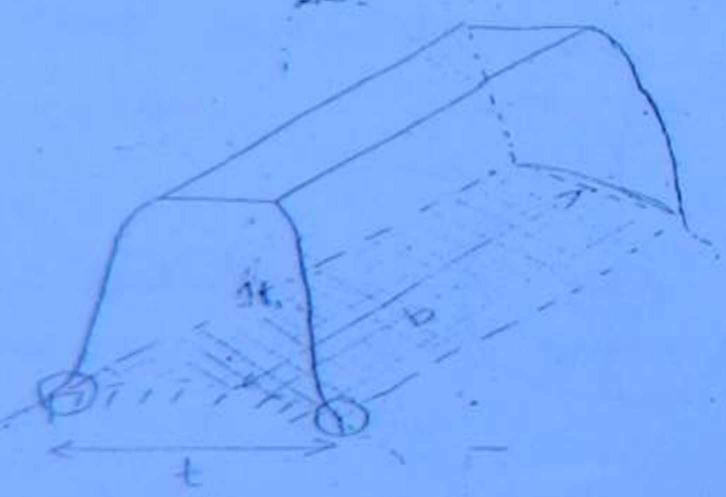
LEWIS (OR) BEAM STRENGTH EQUATION

Assumptions

i) FR is neglected

ii) Cantilever beam is considered

$$\sigma_{b\,max} = \frac{M_{max}}{Z_{NA}} = \frac{F_t \times h}{bt^2/6}$$





$$Z_{NA} = \frac{I_{NA}}{Y_{max}} = \frac{\frac{bt^3}{12}}{t/2}$$

$$\sigma_b = \frac{6 F_t h}{b t^2}$$

$$\therefore F_t = \sigma_b \cdot b \frac{t^2}{6h}$$ {tgt. force acts on Pitch point}

as $T \uparrow, F_t \uparrow, \Rightarrow \sigma_b \uparrow$ (limit)

$\sigma_b = [\sigma_b] \leftarrow$ permissible stress

Then $F_t \Rightarrow F_{t\,max}$

$$\therefore (F_t)_{max} = [\sigma_b]\, b \cdot Y m$$

C/9

Now $\therefore F_t = \sigma_b \cdot b \frac{t^2}{6h} \times \frac{m}{m}$

$$F_t = \sigma_b \cdot b \cdot \left(\frac{t^2}{6hm}\right) m$$

$$F_t = \sigma_b \cdot b \cdot Y \cdot m$$

$$Y = \frac{t^2}{6 \cdot h \cdot m} = \text{Lewis form factor}$$

$$\therefore (F_t)_{max} = [\sigma_b] \cdot b \cdot Y \cdot m \text{ or } F_s$$

$$\boxed{F_s = [\sigma_b] \cdot b Y \cdot m}$$

Beam strength is defined as the Maximum value of Ft that a given gear tooth can withstand without any bending failure (ie. at any instant the load comming on the gear tooth should be less than or equal to Fs to avoid bending failure.

$F_{S1} = 10$ KN (Pinion)

$F_{S2} = 8$ KN (Gear)

ad comming on Gear tooth $\leq (F_S)$ weaker gear

oad comming on Gear tooth $\leq$ [Min of $F_{S1}$ and $F_{S2}$)

$\leq 8$ KN..

oad comming on Gear-tooth

Weaker Gear: (G.G)

$$\boxed{Y \propto Z}$$

$$\Rightarrow Y_2 > Y_1 \quad [\because Z_2 > Z_1]$$

It is a gear which has minimum value of
∴ always we have to design with respect
to a weaker Gear.

When the Gear and pinion are made up of
same material we have to design w.r.t to pinion
only because pinion is the weaker Gear

$[\sigma_{b1}] = [\sigma_{b2}]$, $b_1 = b_2$, $m_1 = m_2$ but $Y_1 < Y_2$ ]

$\therefore F_{S1} < F_{S2}$

When Gears and pinion are made of different
material we have to design with respect to
gear which has minimum value for the
product of $[\sigma_b]$ and Y.

Assumptions Made in Lewis equation

Effect of Fr (ie, axial compressive stress)
are neglected

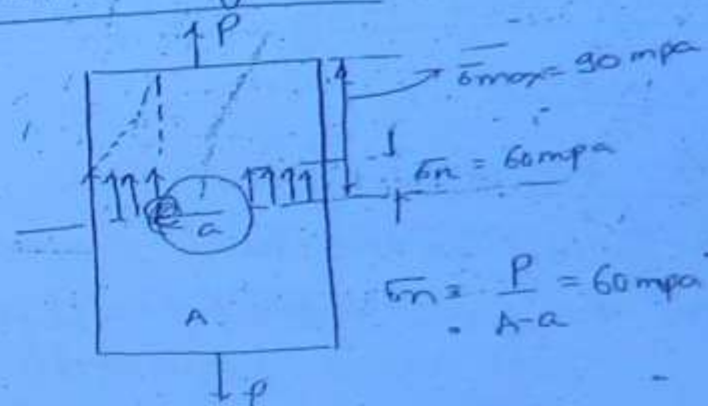
2) Each gear tooth is treated as a cantilever beam fixed at the root portion and free at the tip of the tooth.

3) Effect of stress concentration at the root of the Gear tooth is neglected.

$\Rightarrow \sigma_{max} = K_t$ or $K_f [\sigma_n]$ ($\sigma_n$ = nominal stress)

↳ by using SOM equations

$K_t$ = theoritical stress concentration factor




$$K_t = \frac{\sigma_{max}}{\sigma_n} = \frac{\text{Max stress near the discontinuity}}{\text{Nominal stress}}$$

$$K_t = \frac{90}{60} = 1.5$$

used in static loading

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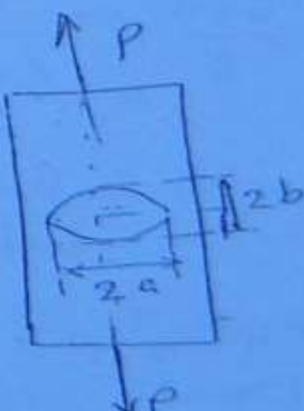
$$\boxed{K_f = 1 + q\,(K_t - 1)}$$

$K_f$ = fatique stress concentration factor

q = notch sensitivity factor Index

for elliptical hole:-




$$\boxed{K_t = 1 + 2\left(\frac{a}{b}\right)}$$

a = Semi major axis

b = Semi minor axis.

for Circular hole $\Rightarrow \boxed{(K_t)_{max} = 3}$

actual values lies between 1 to 3

Because of effect of stress concentration in static loading is less sorter than fatigue loading.

ii) Effect of errors in tooth profiles and tooth spacing are neglected

Effect of Manufacturing errors. (52)

i) contact ratio is assumed as 1. (i.e, it is assumed that only one pair of Gear tooth is in contact

## Dynamic Load ($F_d$)

- It is defined as the load comming on the the gear tooth any instant under dynamic Load

$$(F_d)_{Lewis} = F_t \cdot C_v \quad \text{or} \quad \frac{F_t}{C_v}$$

$C_v$ = velocity factor

$$C_v = \frac{3+v}{3} \quad \text{or} \quad \frac{3}{3+v} \quad [v \leq 10\ m/s]$$

v = pitch line velocity

$$C_v = \frac{6+v}{6} \quad \text{or} \quad \frac{6}{6+v} \quad [10 \leq v \leq 20\ m/s]$$

$$v = v_1 \text{ or } v_2 = \frac{\pi D_1 N_1}{60 \times 1000} \quad \text{or} \quad \frac{\pi D_2 N_2}{60 \times 1000}$$

$$F_t = \frac{2T_1}{D_1} \quad \text{or} \quad \frac{2T_2}{D_2}$$

To avoid bend failure

$$(F_d)_L \leq (F_s)_{\text{weaker Gear}}$$

Buckingham Dynamic Load ($F_d$) (53)

$$F_d = F_t + F_i$$

$$F_d = F_t + \frac{20.67\, v\,[bc + F_t]}{20.67\, v + \sqrt{bc + F_t}}$$

m/s, mm, N/mm

b = facewidth
c = constant

$$c = \frac{e}{K\left[\frac{1}{E_1} + \frac{1}{E_2}\right]}$$

e = error in tooth action in mm
K = constant

Values of e & K are obtained from the tables of design data book.

$F_d \leq F_s \Rightarrow$ Design is safe w.r.t to bending

Reasons for dynamic Load

1) deflection of tooth under load.
2) Inaccuracies in tooth profile
3). Errors in tooth spacing.
4). Misalignment between bearings.
5). Inertia of reciprocating parts

WEAR STRENGTH : [$F_w$]

it is always calculated w.r.t to pinion because pinion is subject to more wear than Gear

[$\because N_1 > N_2$]

$F_w = D_1 \cdot Q \cdot K \cdot b$ in N

$$\boxed{Q = \frac{2G}{G \pm 1}}$$

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⇒ $G$ = Gear Ratio $= \frac{N_1}{N_2}$ {always more than 1}

⇒ + ⇒ for External Gears

⇒ − ⇒ for Internal Gears

b = face width

$$\boxed{K = \text{constant} = \frac{(\sigma_{es})^2 \sin\phi \left[\frac{1}{E_1} + \frac{1}{E_2}\right]}{1.4}}$$

$\sigma_{es}$ = surface endurance limit or Surface fatigue limit

$\phi$ = pressure angle

When $F_d \le F_w$ ⇒ No wear failure

for Safe designing Gear

$$\boxed{F_w > F_s > F_d}$$

$F_w > F_s$ generally.

DESIGN PROCEDURE USED IN SPUR GEAR

Data: $P = x$ kW at $Y$ RPM

$G$ = Given [Gear ratio]

$G$ or $N_2$ will be given

$\phi$ ; $[\sigma_{b1}]$, $[\sigma_{b2}]$, $\sigma_{es}$,

(1) $G = \frac{N_1}{N_2} = \frac{\text{speed of pinion}}{\text{Speed of Gear}} = ?$

(2) $G = \frac{N_1}{N_2} = \frac{D_2}{D_1} = \frac{Z_2}{Z_1}$

$Z_2 = G \cdot Z_1$

imp. (3) $\boxed{Z_1 \geq (Z_1)_{min}}$

$(Z_1)_{min}$ = minimum no. of teeth provided on the pinion to avoid interference

$$\boxed{(Z_1)_{min} = \frac{2 \cdot a_w}{\sin^2 \phi}}$$

$a_w$ = addendum coefficient

$$\boxed{a_w \times m = a}$$

⇒ For full depth tooth ⇒ $a = m \Rightarrow a_w = 1$

⇒ for stub tooth ⇒ $a = 0.8m \Rightarrow a_w = 0.8$

⇒ for $\phi = 20°$ (Full depth).

$$(Z_1)_{min} = \frac{2 \times 1}{\sin^2 20°} = 17.09$$

$$Z_1 = 18$$

⇒ for $\phi = 20°$ (stub teeth)

$$Z_{1\,min} = \frac{2 \times 0.8}{\sin^2 20°} = 13.67$$

$$Z_1 = 14$$

$$Z_2 = G \cdot Z_1$$

$$T_1 = \frac{\overset{kW}{P} \times 60}{2\pi N_1} \times 10^6 = \_\_\_ \text{ N-mm}$$

$[T_1]$ = design torque = $T_1 \cdot K_a$ = ___ N-mm

$K_a$ = overload / service factor

$K_a = 1.25$

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(Module :-)

$$\boxed{m = 1.26 \sqrt[3]{\frac{[T_1]}{([\sigma_b] \cdot Y)_{w \cdot G} \, \psi \cdot Z_1}}}$$

$$\psi = \frac{b}{m} = 10 \Rightarrow \boxed{b = 10\,m}$$

$$8 \le \psi \le 12$$

$$F_S = (F_t)_{max} = ([\sigma_b] \cdot Y)_{w \cdot G} \, b \cdot m$$

$$\frac{2[T_1]}{D_1} = ([\sigma_b] Y)_{w \cdot G} \, \psi m m$$

$$\frac{2[T_1]}{m Z_1} = ([\sigma_b] Y)_{w \cdot G} \, \psi m^2$$

$$2[T_1] = ([\sigma_b] Y)_{wG} \, \psi \cdot m^3 Z_1$$

$$m^3 = \frac{2[T_1]}{([\sigma_b] Y)_{wG} \, \psi \cdot Z_1}$$

$$\therefore m = \sqrt[3]{\frac{2[T_1]}{([\sigma_b] Y)_{w \cdot G} \, \psi Z_1}}$$

$$\therefore \boxed{m = 1.26 \sqrt[3]{\frac{[T_i]}{([\sigma_b]Y)_{wk}\,\psi \cdot Z_1}}}$$

(6) Dimension of Gear tooth

$D_1 = m \cdot Z_1$

$D_2 = m \cdot Z_2$

57

$C = \frac{D_1 + D_2}{2} = \frac{m}{2}[Z_1 + Z_2]$

$a = m$

$b = \psi m = 10m$

$d = 1.157m$

$c = d \sim a$

Q. A pair of spur gears having $14\frac{1}{2}^\circ$ involute full depth teeth is to transmit 12 kW at 300 RPM the Gear ratio is 3:1 the static strength of the pinion the CI Gear and steel pinion are 60 MPa and 105 MPa respectively design the Gear pair, checks for dynamic strength and wear strength by assuming following data $C_v$ is $C_v = \frac{4.5}{4.5+v}$, $\sigma_{el} = 600$ MPa, $E_{MS} = 200$ GPa

$E_{CI} = 100$ GPa Ans.

Data: $\phi = 14\frac{1}{2}^\circ$, $P = 12$ kW

$N_1 = 300$ RPM, $G = 3:1$

$\sigma_{b1} = 105$ mPa, $\sigma_{b2} = 60$ MPa

$G = \frac{N_1}{N_2} = \frac{Z_2}{Z_1} = 3$

$Z_2 = 3Z_1$

) $z_1 \geq (z_1)_{min}$

$$(z_1)_{min} = \frac{2a_w}{\sin^2\phi}$$

for full depth pinion $a_w = 1$

$$(z_1)_{min} = \frac{2\times1}{\sin^2 14\frac{1}{2}} = 31.9$$

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$z_1 \geq 31.9$

$z_1 = 32$, $z_2 = 3\times z_1 = 96$

) $T_1$ = torque to be transmitted by the pinion

$$T_1 = \frac{P \times 60}{2\pi N} \times 10^6 \quad (P\text{ in kW})$$

$$= \frac{12\times60}{2\pi\cdot300}\times10^6 \doteq 381.97\times10^3 \text{ N-mm}$$

) Design Torque = $[T_1]$

$[T_1] = T_1 \times k_a$

assuming overload as 25%

$\therefore k_a = 1.25$

$[T_1] = 381.97\times10^3 \times 1.25$

$[T_1] = 477.46\times10^3$ N-mm

$$m \geq 1.26 \sqrt[3]{\frac{[T_1]}{([\sigma_b]y)_{wk}\ \psi\ z_1}}$$

$[\sigma_{b1}]y$ & $[\sigma_{b2}]y_2$ = ?

$\phi = 14\frac{1}{2}$ (FD) $y = 0.124 - \frac{0.684}{z}$

$\phi = 20° (FD)$

$$y = 0.154 - \frac{0.912}{z}$$

$\phi = 20° (stub)$

$$y = 0.175 - \frac{0.841}{z}$$

$$y_1 = 0.124 - \frac{0.684}{z_1} = 0.10265$$

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$$y_2 = 0.124 - \frac{0.684}{z_2} = 0.1168$$

$$Y_1 = \pi y_1 = \pi \times 0.10265 = 0.322$$

$$Y_2 = \pi y_2 = \pi \times 0.1168 = 0.367$$

$[\sigma_{b1}] Y_1 = 33.85$ mpa.

$[\sigma_{b2}] Y_2 = 22.03$ MPa

$[\sigma_{b2}] Y_2 < [\sigma_{b1}] Y_1$

$\Rightarrow$ Gear is weaker

Hence we have to design w.r.t Gear

$$[[\sigma_b] Y]_{WG} = [\sigma_{b2}] Y_2 = 22.03 \text{ mpa}$$

$$\psi = \frac{b}{m} = 8 \text{ to } 12$$

$$b = 10 m$$

$$m \geq 1.26 \sqrt[3]{\frac{477.46 \times 10^3}{22.03 \times 10 \times 32}}$$

$\therefore m \geq 5.136$

$\underline{m} = 6$ mm

i) Dimension of Gear pair

$D_1 = mZ_1 = 192$ mm

$D_2 = mZ_2 = 576$ mm

$c = \frac{m}{2}[Z_1 + Z_2] = < - 384$ mm

66

$b = 10m = 60$ mm

$p_c = \pi m = 18.85$ mm

$a = m = 6$ mm

$d = 1.157\,m = 6.94$ mm

i) Beam strength

$(F_s)_{WG} = F_s([\sigma_b]Y)_{wr}\, b\, m$

$= 22.03 \times 60 \times 6$

$= \cancel{1321.8\,N}\ 7930.8\,N$

i) Check for dynamic load w.r.t to bending failure

$(F_d)_L = F_t \times C_v$ or $\frac{F_t}{C_v}$

$C_v = \frac{4.5}{4.5+v} = 0.5987$

$v = \frac{\pi D_1 N_1}{60\times1000} = \frac{\pi \times 192 \times 300}{60\times1000} = 3.016$ m/s

$F_t = \frac{2T_1}{D_1}$ or $\frac{2T_2}{D_2}$

$F_t = \cancel{4.91 \times 10^3\,kN}\ 3978.85$ N

$$(F_d)_L = \frac{F_t}{C_v} = \frac{4.97\times10^3}{0.5987} = 6645.8\ N$$

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$\therefore$ Since $(F_d)_L < F_s$

design of Gear pair is safe with respect to bending failure

(9) <u>check for wear failure or wear strength</u>

wear strength is always calculate for pinion as pinion is subjected to more wear

$$F_w = D_1 \cdot Q \cdot K b$$

$$Q = \frac{2G}{G\pm1} = \frac{2\times3}{3+1} = 1.5$$

$$K = \frac{\sigma_{es}^2 \cdot \sin\phi}{1.4}\left[\frac{1}{E_1}+\frac{1}{E_2}\right] = \frac{(600)^2\times\sin14.5}{1.4}\left[\frac{1}{200\times10^3}+\frac{1}{100\times10^3}\right]$$

$$\therefore K = \cancel{161\times10^3}\ 0.965 = K$$

$$F_N = 192\times1.5\times0.965\times60$$

$$F_W = 16675.2\ N$$

$F_W > F_d \Rightarrow$ design is safe for Gear pair w.r.t to wear failure

Q. 2002 IES

## ③ DESIGN OF SHAFTS

axle : subjected to only bending

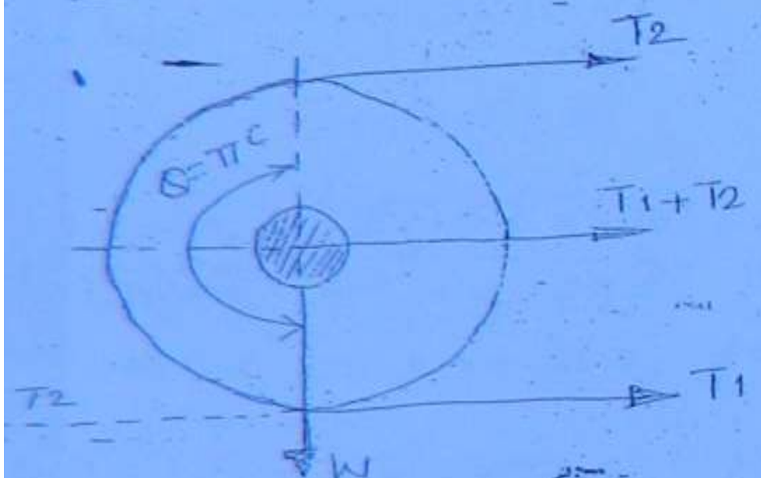
spindle : a short rotating shaft which supports tool or w/p

(62)

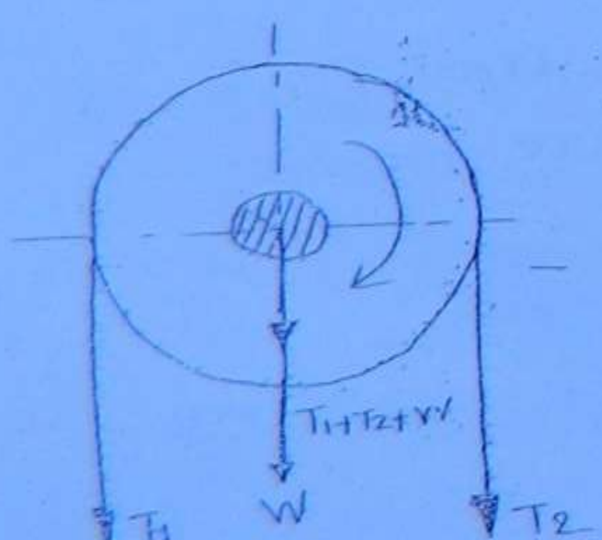
① $T = \frac{P \times 60 \times 10^6}{2\pi N} = \_\_ \; N\cdot mm$ (P → kW, N → RPM)

② Shafts with pulleys :-

(a) Horizontal belt drive




(b) Vertical belt drive



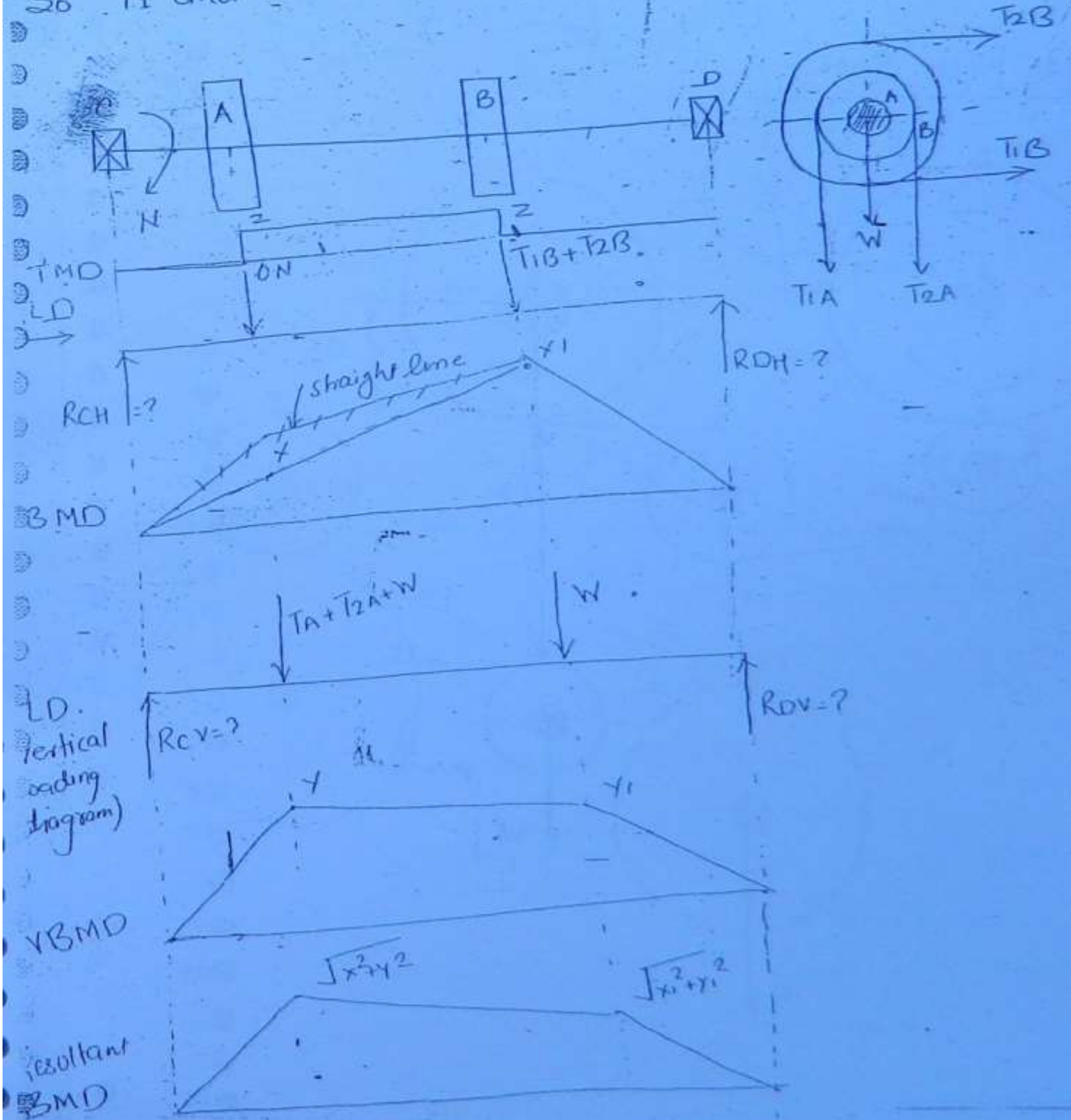
$\boxed{T = (T_1 - T_2) R}$ → V. Imp.

$T_1 - T_2 = \overset{?}{\text{—}}$ (I)

$\dfrac{T_1}{T_2} = e^{\mu\theta} = ✓$ (II)

$T_{max} = \sigma_{per} \cdot b \cdot t = \text{—}$ (III)   (G3)

So $T_1$ and $T_2$ can be found out.



## Design of shafts

as shaft is subjected to both BM and TM we have to go for Theories of failure

$$T_e = \sqrt{(M_R)_A^2 + T_A^2} = \frac{\pi}{16} d^3 \cdot \tau_s$$

$\therefore d = ?$

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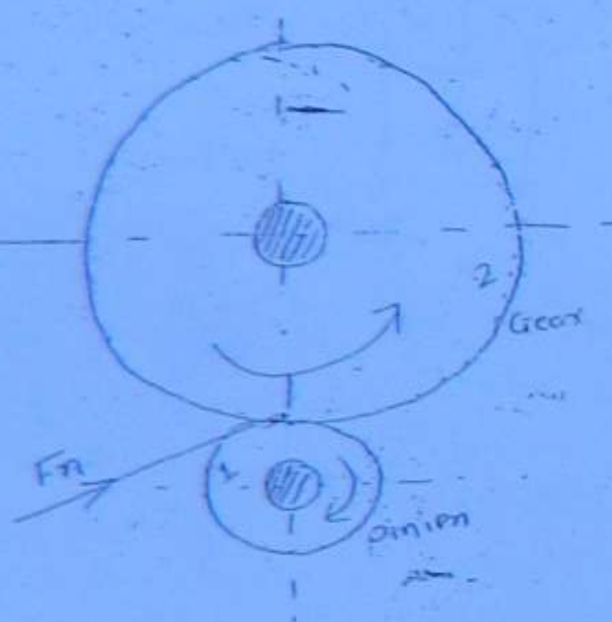
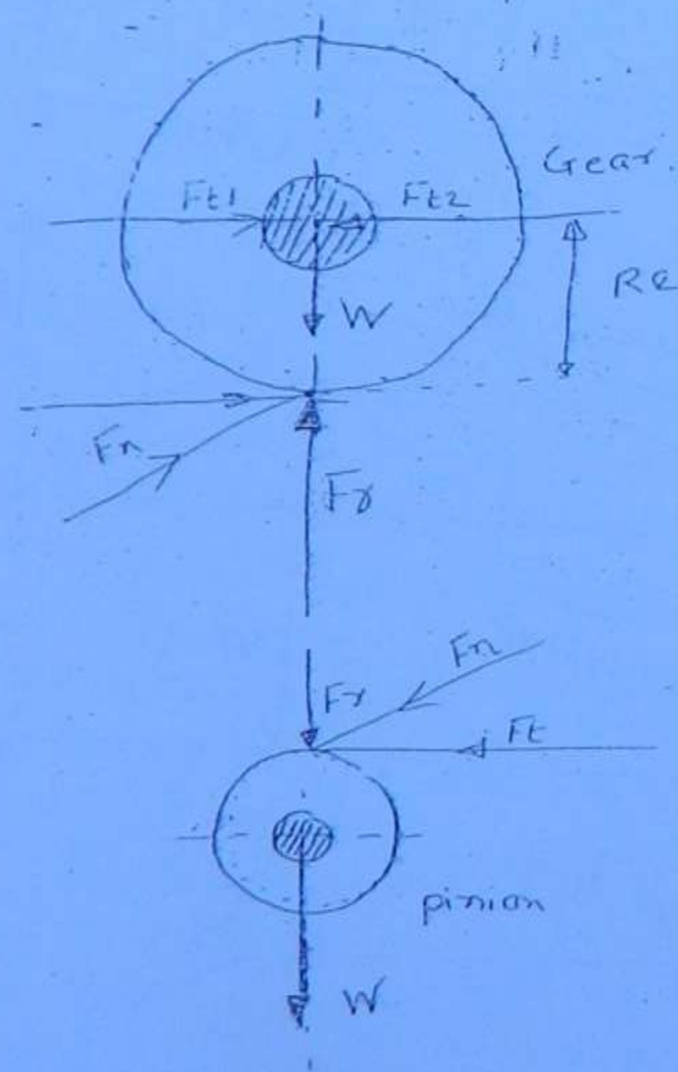
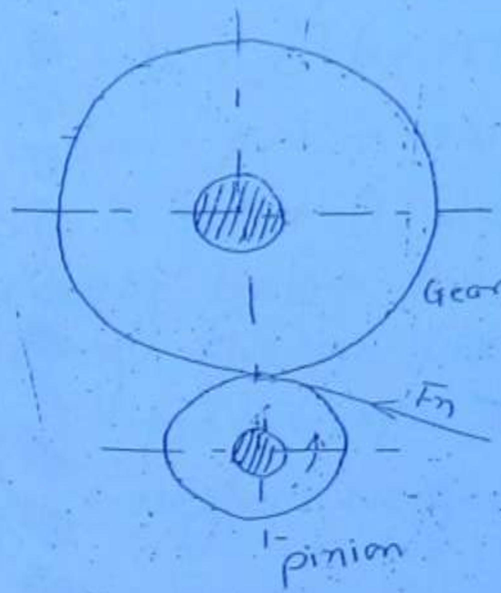
### i) Shafts with Gears

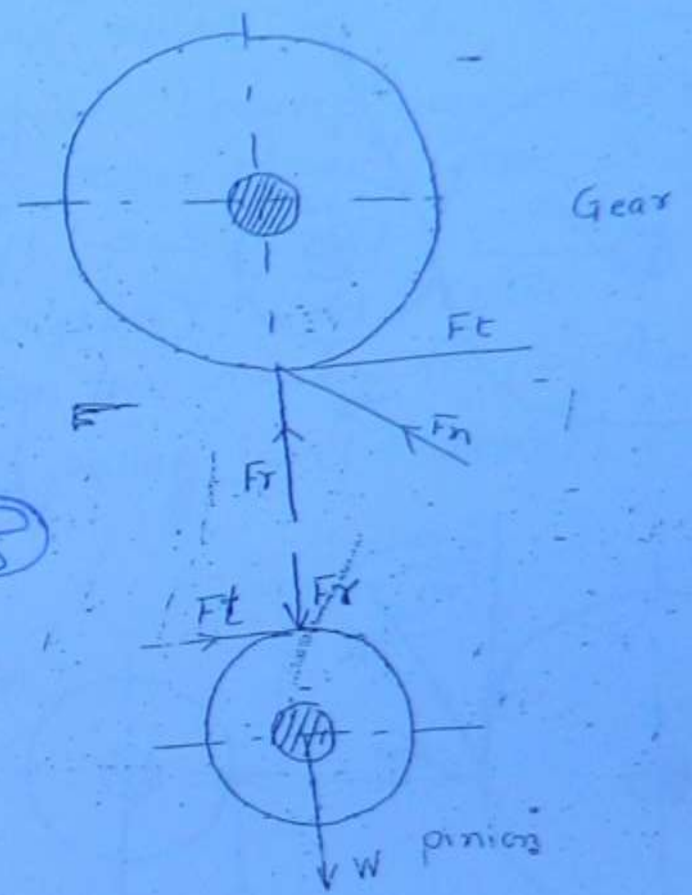



11.

(2)

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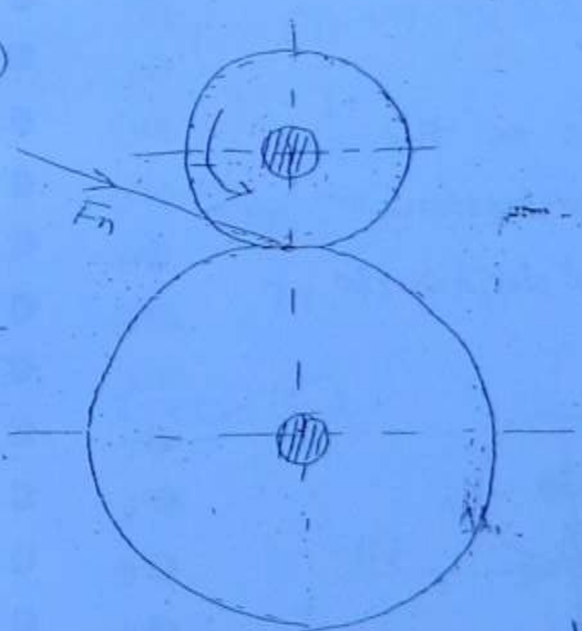
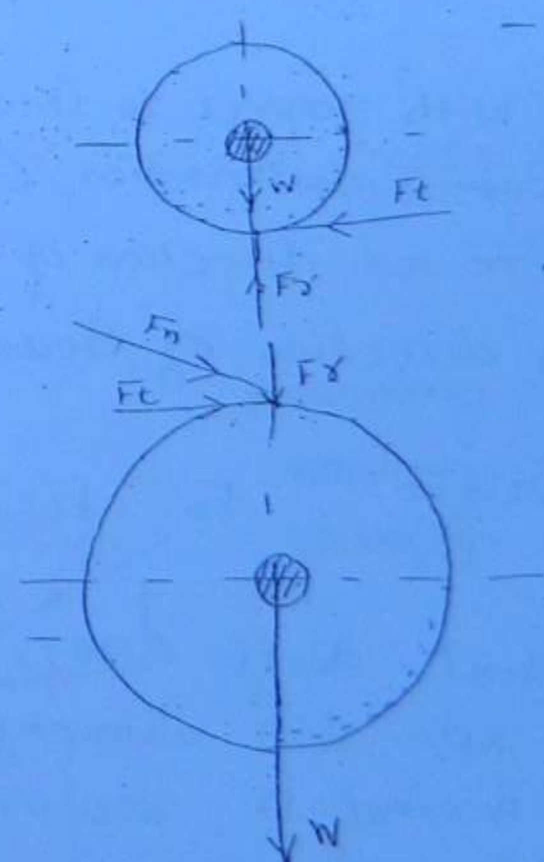
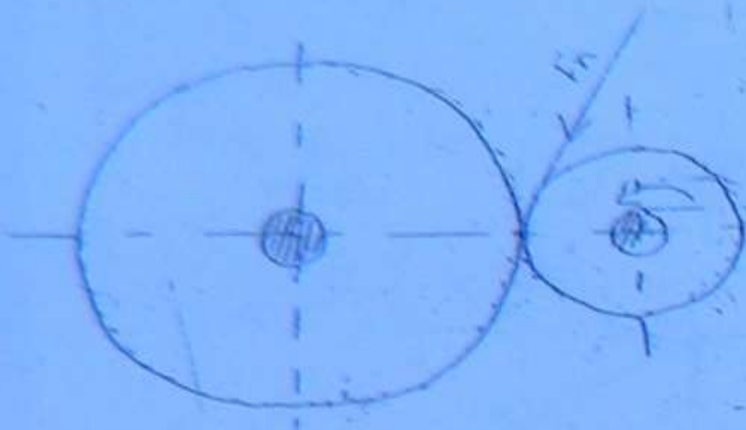
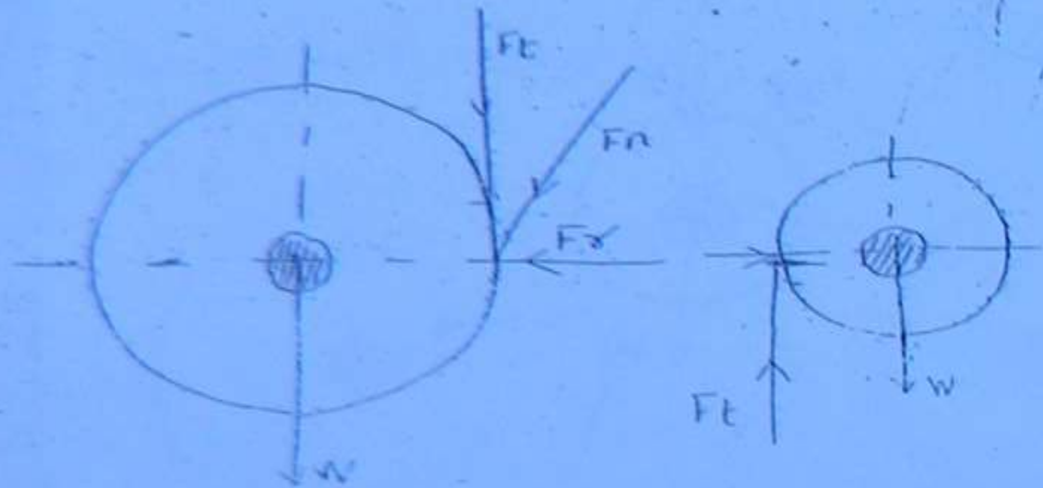
(3)






(66)




direction of Fr with respect to Gear is in the direction of power Transmission (Top to bottom or left to right) where as direction of Ft depends on the rotation direction of Gear.

$$F_t = \frac{2T_1}{D_1} \text{ or } \frac{2T_2}{D_2} \qquad F_r = F_t \tan\phi$$

In a Mild steel shaft ABCD Transmits 20 KW at 300 RPM, it is simply supported on bearings at A and D, 800 mm apart it carries a pulley of 500 mm diameter located at a point B (AB = 200 mm) which recieves power, by a horizontal belt drive with the belt tension ratio of 2, 200mm diameter, 20° involute gear Located at point 'C' (CD is 200 mm), delivers power to a Gear directly below the shaft, assuming safe working stresses ($\sigma_t$ = 70 mpa) and $\tau_s$ is equal to 56 MPa, design the diameter of the shaft (Neglect weight of pulley and Gear).

(67)

Soln (pulley B)

P = 20 KW, N = 300 RPM

$$T = \frac{P \times 60}{2\pi N} \times 10^6$$

$$T = 636619.77 \text{ N-mm}$$

$$T = (T_1 - T_2) \times 250$$

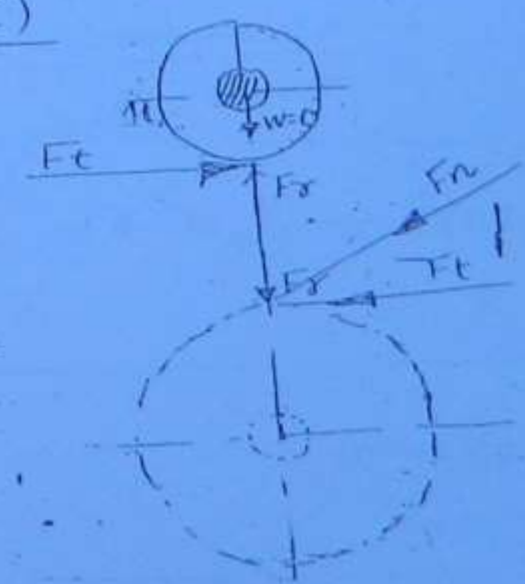
$$\frac{T_1}{T_2} = 2$$

$T_1 + T_2 = 7639.44 N$

$T_1 = 5092.96 N$

$T_2 = 2546.48 N$

Gear (C)




Assuming shaft rotating in clockwise direction

$T_C = T_B = 636619.17 = F_t \cdot R_c$

$F_t = 6366.19\ N$

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$F_r = F_t \tan\phi = 2317.11\ N$

$$(M_R)_B = \sqrt{(HBM)_B^2 + (VBM)_B^2} = 1468775.97\ N\cdot mm$$

$$(M_R)_C = \sqrt{(1336900)^2 + (347550)^2} = 1381337.25\ N\text{-}mm$$

⇒ The critical point is B on shaft because MR is Max and Torque is maximum; hence we have to design w.r.t 'B' on shaft.

Design of shaft

i.e. Shaft is design using Theories of failures (MSST and MDET) becouse it is subjected to both BM and TM.

MSST $$(T_e)_B = \sqrt{(k_b M_R)_B^2 + (k_t T_B)^2} = \frac{\pi}{16} d^3 \tau_s$$

Where $k_b$ and $k_t$ combined shock and fatigue factor for bending, combined shock and fatigue factor for Twisting respectively.

$k_b = 1.5$ and $k_t = 2$ ← Assumption

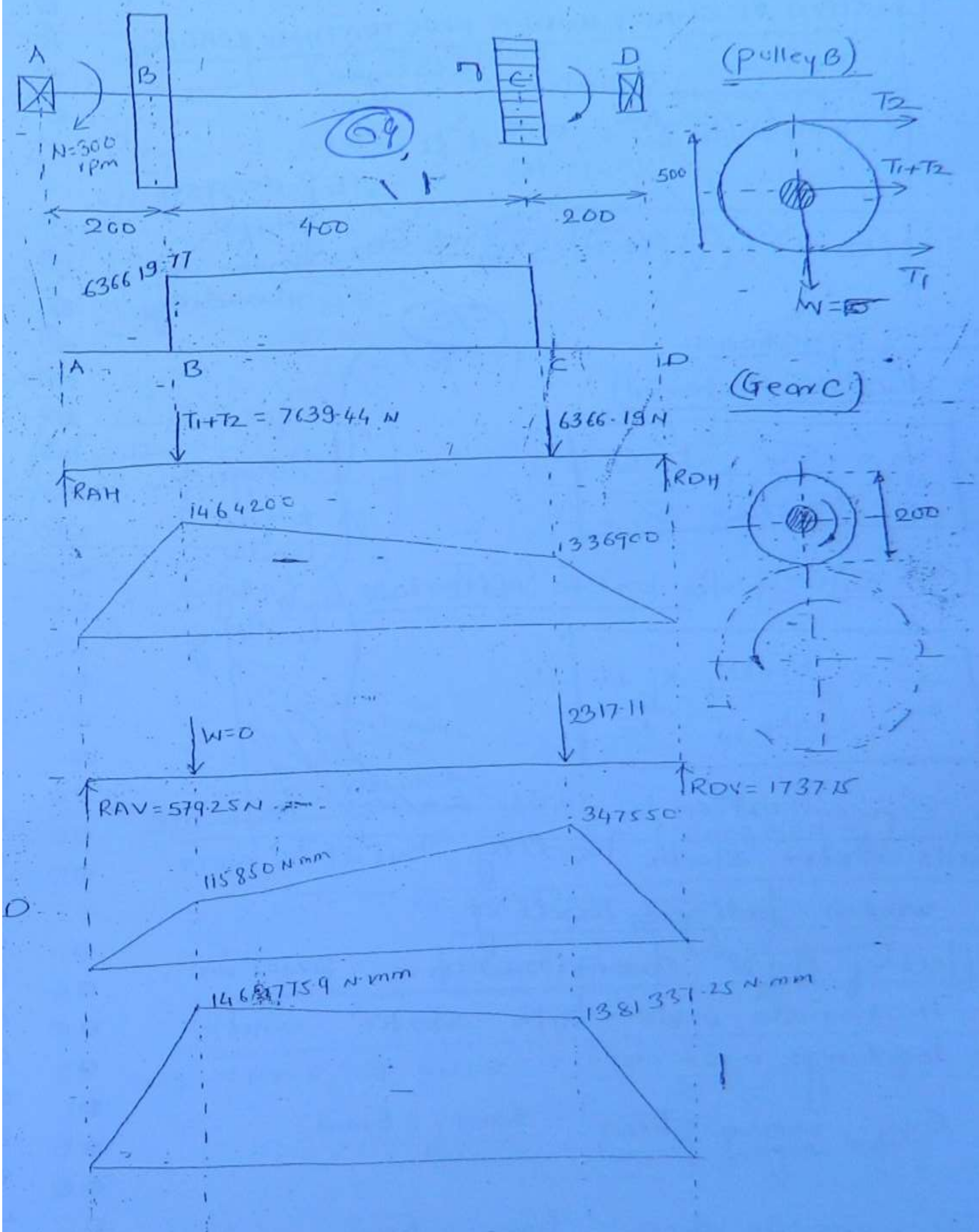
$d = 58.5$ mm

MDET $$(M_e)_B = \sqrt{(k_b M_R)_B^2 + \frac{3}{4}(k_t T_B)^2} = \frac{\pi}{32} d^3 \cdot \sigma_b$$

$\therefore d = 71.03$ mm

$\therefore$ choose $d = 71.03$ mm

$\therefore d = 75$ mm


A
B
C
D
(pulley B)
64
N=300 rpm
200
400
200
T2
T1+T2
500
T1
W=
636619.77
A
B
C
D
(Gear C)
T1+T2 = 7639.44 N
6366.19 N
RAH
RDH
200
1464200
1336900
2317.11
W=0
RAV=579.25 N
RDV= 1737.75
347550
115850 N mm
1468?759 N-mm
1381337.25 N-mm

# DESIGN OF SHAFT UNDER FLUCTUATING LOADING
(fatigue)

$$T_e = \sqrt{(K_b \cdot M)^2 + (K_t \cdot T)^2} = \frac{\pi}{16} d^3 \tau_s$$

$$M_e = \sqrt{(K_b \cdot M)^2 + \left(\frac{3}{4}[K_t \cdot T]^2\right)} = \frac{\pi}{32} d^3 \sigma_b$$

designing shaft under static Loading

$\sigma_t$ / $\sigma_b$

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Soderberg equations:-
(for ductile Material)

$$\boxed{\frac{1}{N} = \frac{\sigma_m}{\sigma_{yt}} + \frac{K_f \cdot \sigma_v}{\sigma_e}}$$

$\sigma_{yt} = S_{yt}$

Goodman Eq^n → for brittle Materials

$$\boxed{\frac{1}{N} = \frac{\sigma_m \cdot K_t}{\sigma_{ut}} + \frac{K_f \cdot \sigma_v}{\sigma_e}}$$

$\sigma_{ut} = S_{ut}$

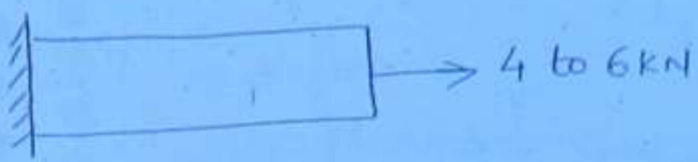
designing torque under fatigue Loading

Stress concentration is less serious ~~series~~ in ductile materials under static loading but it is more series under fatigue loading.

Effect of stress concentration is sirios in brittle materials under both static and fatigue loadings

$$\sigma_m = \text{mean stress} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_v = \text{variable stress} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

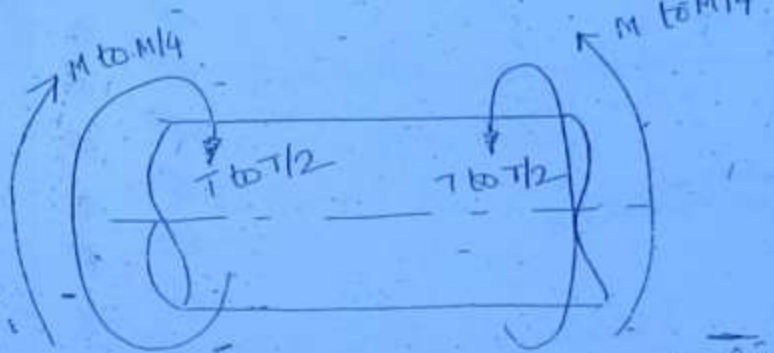


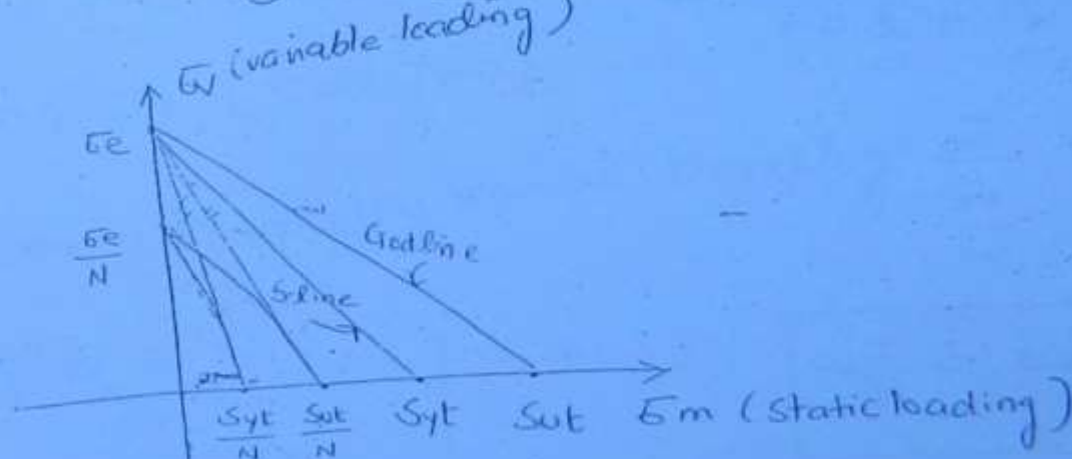

$\sigma_m \neq 0$
$\sigma_v \neq 0$ } alternating loads

$$\sigma_{max} = \frac{P_{max}}{\pi/4 \cdot d^2} = \frac{6000}{\frac{\pi}{4} d^2}$$

$$\sigma_{min} = \frac{P_{min}}{\frac{\pi}{4} \cdot d^2} = \frac{4000}{\frac{\pi}{4} d^2}$$

7




$\Rightarrow$ X-axis, $\Rightarrow \sigma_v = 0 = \frac{\sigma_{max} - \sigma_{min}}{2}$

$\boxed{\sigma_{max} = \sigma_{min}}$ Static loading

$\Rightarrow$ Y axis, $\Rightarrow \sigma_m = 0, = \frac{\sigma_{max} + \sigma_{min}}{2}$

$\boxed{\sigma_{max} = -\sigma_{min}}$ fatigue loading

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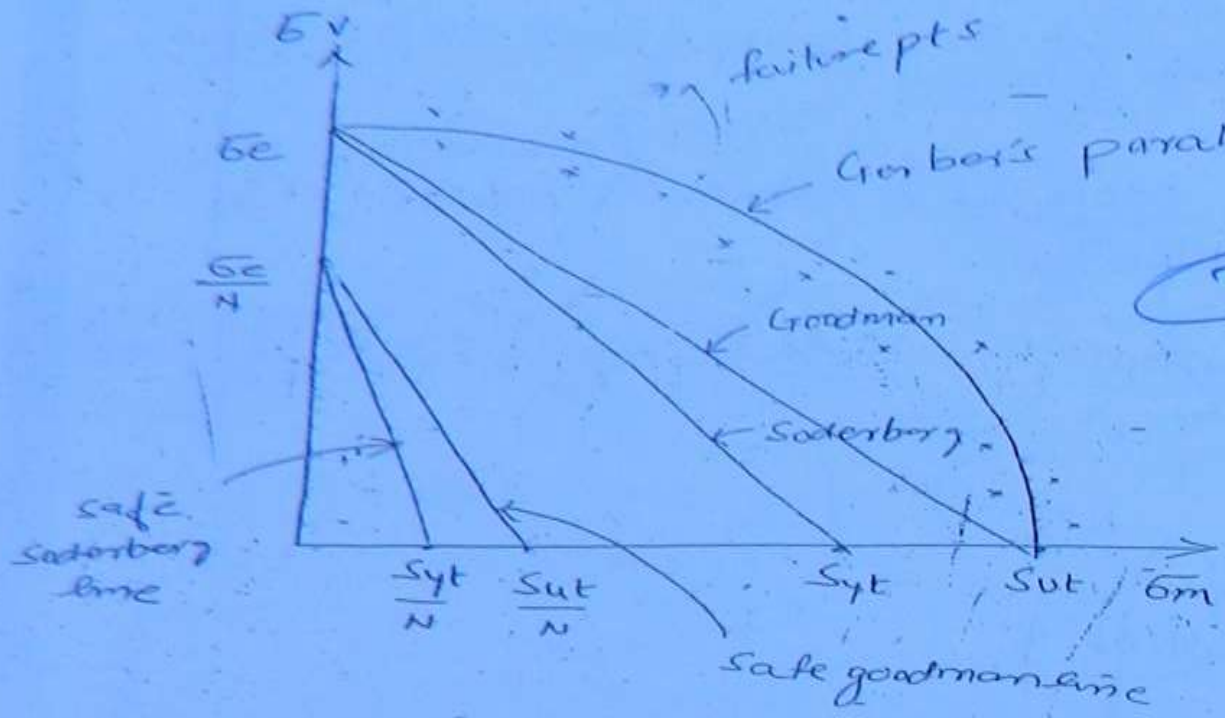


eqn of line

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{\sigma_m}{\frac{S_{yt}}{N}} + \frac{\sigma_v}{\frac{\sigma_e}{N}} = 1$$

$$\boxed{\frac{\sigma_m}{S_{yt}} + \frac{K_f \sigma_v}{\sigma_e} = \frac{1}{N}}$$

Soderberg line - will give safest design

$$\boxed{\sigma_e = \sigma_e^* \; K_a \cdot K_b \cdot K_c}$$

$\sigma_e^*$ = endurance limit of a standard specimen (from fatigue test)

$\sigma_e$ = endurance limit of a Mechanical component

→ $K_a$ = Size factor

→ $K_b$ = Surface finish factor

Size ↑ ⇒ defects ↑

EL ↓ ⇒ Ka ↓

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SF ↓ ⇒ Roughness ↑

EL ↓ ⇒ Kb ↓

$K_c = 1$ → for completey reverse bending

$= 0.7$ → for completely reverse axial loading

$= 0.6$ → for completely reverse torsion

$\sigma_e^* = 0.5\, S_{ut}$ [Steel]

$\sigma_e^* = 0.4\, S_{ut}$ [Cast Iron]

$\sigma_e^*$ = EL under completely reverse bending

$$\frac{1}{N} = \frac{\sigma_m}{\sigma_{yt}} + \frac{K_f\, \sigma_v}{\sigma_e}$$

$$\frac{\sigma_{yt}}{N} = \sigma_m + \frac{K_f\, \sigma_v\, \sigma_{yt}}{\sigma_e}$$

$$\sigma_{eq} = \sigma_m + \frac{K_f\, \sigma_v\, \sigma_{yt}}{\sigma_e} \quad (1)$$

$$\sigma_{eq} = \frac{S_{yt}}{N} = \frac{\sigma_{yt}}{N}$$

used in Axial or Bending

## Torsion

replace $\sigma \rightarrow \tau$

$$\frac{1}{N} = \frac{\tau_m}{\tau_{ys}} + \frac{K_f \cdot \tau_v}{\tau_e}$$

$$\tau_{ys} = S_{ys} = \frac{S_{yt}}{2}$$

$$\tau_e = \underbrace{\sigma_e^*}_{0.5\,S_{ut}} \cdot K_a \cdot K_b \cdot \overset{0.6}{K_c}$$

$$\frac{\tau_{ys}}{N} = \tau_m + \frac{K_f\,\tau_v \cdot \tau_{ys}}{\tau_e} \qquad \tau_e = \frac{S_{ys}}{N}$$

$$\tau_{eq} = \tau_m + \frac{K_f\,\tau\,\tau_{ys}}{\tau_e} \quad \text{—(2)}$$

MSST

$$\sigma_t = \frac{S_{yt}}{N} = \sqrt{\sigma_x^2 + 4\tau_{xy}^2}$$

$$\sigma_t = \frac{S_{yt}}{N} = \sqrt{(\sigma_{eq})^2 + 4(\tau_{eq})^2}$$

$d = ?$

MDET

$$\sigma_t = \frac{S_{yt}}{N} = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

$$\sigma_t = \frac{S_{yt}}{N} = \sqrt{\sigma_{eq}^2 + 3(\tau_{eq})^2}$$

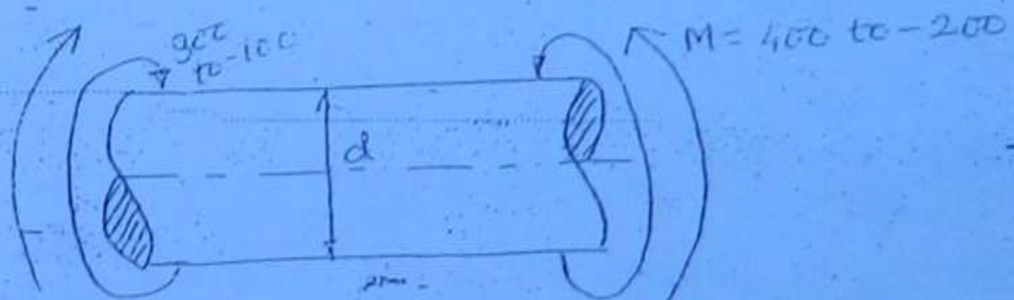
MSST

$$\tau_s = \frac{S_{ys}}{N} = \left(\frac{S_{yt}}{2N}\right) = \frac{1}{2}\sqrt{\sigma_x^2 + 4\tau_{xy}^2}$$

$$\tau_s = \frac{S_{ys}}{N} = \frac{1}{2}\sqrt{\sigma_{eq}^2 + 4\tau_{eq}^2}$$ 575

IES-04

Q: A hot rolled steel shaft is subjected to a torsional load that varies from 300 kN·mm (clockwise) to 100 kN·mm (counterclockwise), As an applying bending moment at a critical section varies from 400 kN·mm to −200 kN·mm, the shaft is of uniform cross-section and no keyway is present at the critical section. determine the reqd shaft diameter by taking factor of safety as 1.5, Sut is 560 mpa, Syt is 420 mpa, design stress is 280 mpa also take the modification factor as 0.62, size correction factor as 0.85, Load factor (Kc) as 1 and load factor for Torsion 0.58.




Bending

$$\sigma_{max} = \frac{M_{max}}{Z} = \frac{32\,M_{max}}{\pi d^3} = \frac{32\times 400\times 10^3}{\pi d^3}$$

$$= \frac{x}{d^3}\ \text{MPa}$$

$$\sigma_{min} = \frac{32\,M_{min}}{\pi d^3} = \frac{32\times -200\times 10^3}{\pi d^3}$$

$$= \frac{y}{d^3}\ \text{MPa}$$

$$\sigma_m = \frac{\sigma_{max}+\sigma_{min}}{2} \qquad \sigma_a = \frac{\sigma_{max}-\sigma_{min}}{2}$$

$K_f = 1$

$\sigma_{yt} = 420\ \text{Mpa}$ , $\sigma_e = \sigma_e^* \cdot K_a \cdot K_b \cdot K_c$ 76

$= 280 \times 0.85 \times 0.62 \times 1$

$\sigma_e = ?\ \text{Mpa}$

by using soderberg eqn

$$\sigma_{eq} = \sigma_m + \frac{K_f \sigma_v \sigma_{yt}}{\sigma_e} = \frac{X^*}{d^3}\ \text{MPa} \longrightarrow (1)$$

Torsion case

$T_{max} = 800 \times 10^3\ \text{N.mm}$

$T_{min} = -100 \times 10^3\ \text{N-mm}$

$$\tau_{max} = \frac{T_{max}}{Z_p} = \frac{16\, T_{max}}{\pi d^3} = \frac{?}{d^3}\ \text{Mpa}$$

$$\tau_{min} = \frac{16\, T_{min}}{\pi d^3} = \frac{?}{d^3}\ \text{Mpa}$$

$$\tau_m = \frac{\tau_{max} + \tau_{min}}{2} = \frac{?}{d^3}\ \text{Mpa}$$

$$\tau_v = \frac{\tau_{max} - \tau_{min}}{2} = \frac{?}{d^3}\ \text{Mpa}$$

by using soderberg equation

$K_f = 1$ , $\tau_{ys} = \frac{\sigma_{yt}}{2} = 210\ \text{Mpa}$

$\tau_e = \sigma_e^* \cdot K_a \cdot K_b \cdot K_c =$

$= 280 \times 0.85 \times 0.62 \times 0.58$

$=$ ___ mpa

$$\tau_{eq} = \tau_m + \frac{K_f \tau_v \tau_{ys}}{\tau_e} = ?(Y^*)\ \text{MPa} \longrightarrow (2)$$

by using

MDET $\sigma_t = \frac{S_{yt}}{N} = \sqrt{(\sigma_{eq})^2 + 3(\tau_{eq})^2}$

'd' can be found out

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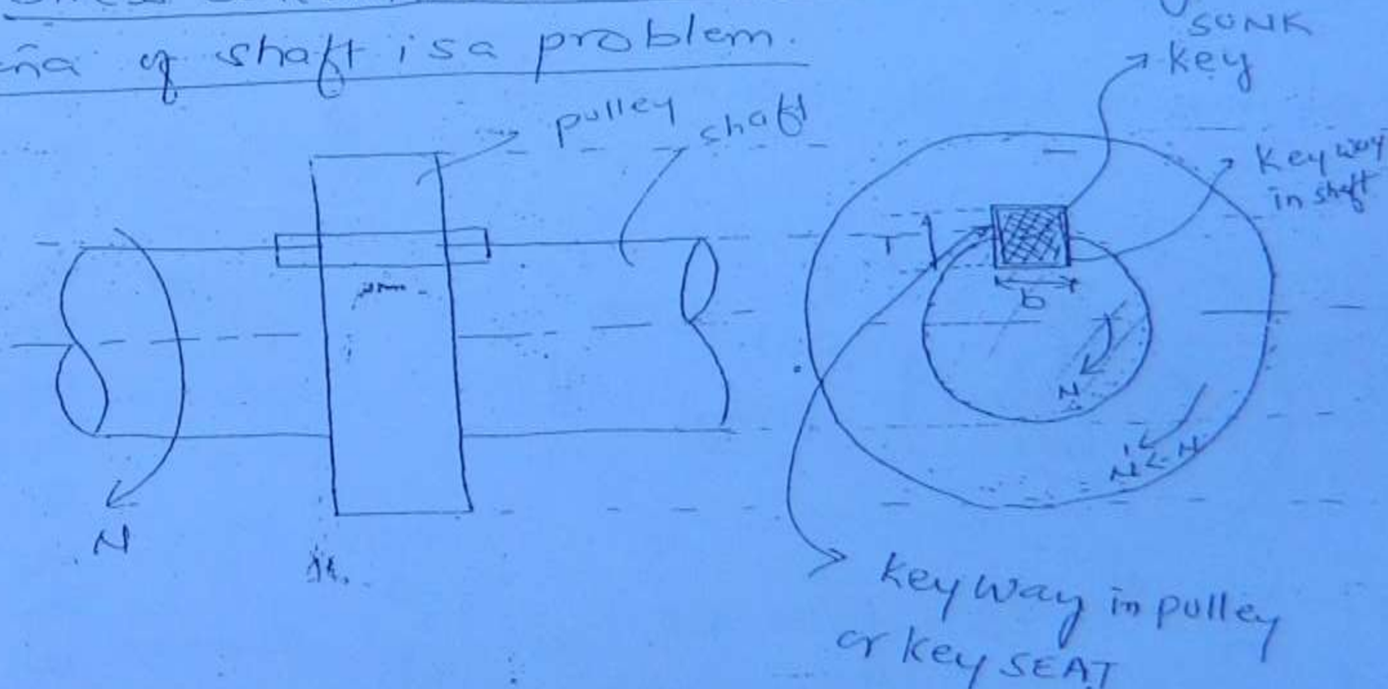
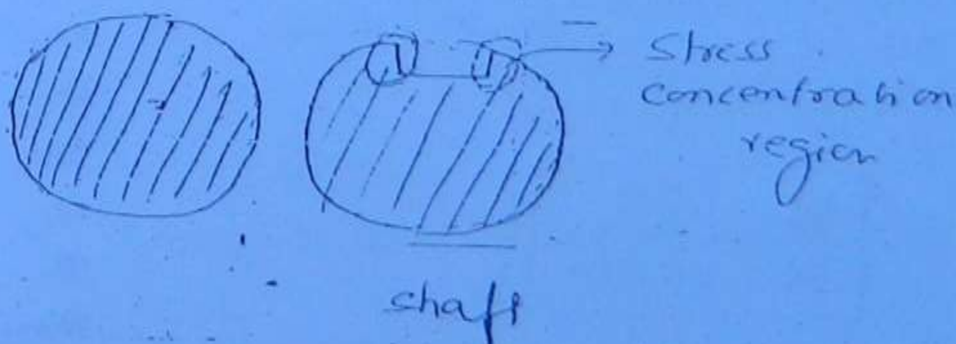
## 4 KEYS

Keys is defined as a metal piece which is inserted between shaft and its assembly to transmit power between them and to prevent relative Motion between them.

Key acts as a safety device for shaft and its assembly in presence of overloads.

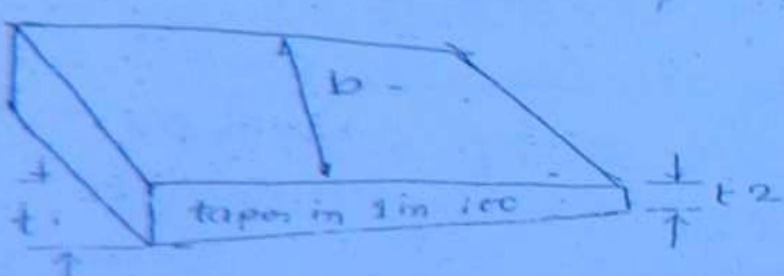
Stress concentration factor and strength criteria of shaft is a problem.



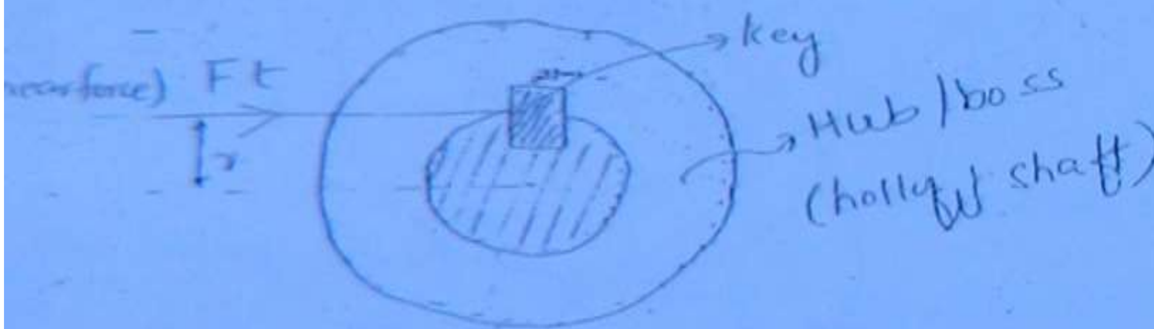



3D figure of key


(i) parallel key




every 100 mm in length thickness reduces by 1mm




$F_t = \frac{2T}{d}$ , $\tau_s = \frac{F_s}{A_s} = \frac{F_t}{L \times b} = \frac{2T}{d l b}$

d = diameter of shaft

$\tau_{ind} \leq \tau_{per}$

$$\boxed{\tau_s = \frac{2T}{d \cdot l \cdot b}}$$

$$\frac{2T}{d \cdot l \cdot b} \le \tau_{per}$$

$$b \ge \_\_ \text{ mm}$$

Crushing stress ($\sigma_c$)

$$\sigma_c = \frac{F_t}{A_c} = \frac{2T}{d \times l \frac{t}{2}}$$

$$\therefore \boxed{\sigma_c = \frac{4T}{d \cdot l \cdot t}}$$

$$\therefore \sigma_c \le (\sigma_c)_{permissible}$$

$$\therefore t \ge \frac{?}{} \text{ mm}$$

<u>Standard propotion of key</u>

$U = d + 13$

$L = 1.5\,U$

$b = \frac{U}{4}$ $\quad t = \frac{U}{6}$

$\rightarrow U = 4b$ $\quad \therefore t = \frac{4b}{6} = \frac{2}{3} b$

$$\therefore \boxed{t = \frac{2}{3} b}$$

d = diameter of shaft

<u>check for safe design</u>

$$\tau_{ind} = \frac{2T}{d l b} = ?$$

$\tau_{ind} \le \tau_{per}$ [No shear failure]

$(\sigma_c)_{ind} = \frac{4T}{d\,l\,t} = ?$

$(\sigma_c)_{ind} \le (\sigma_c)_{permissible}$

So increase thickness

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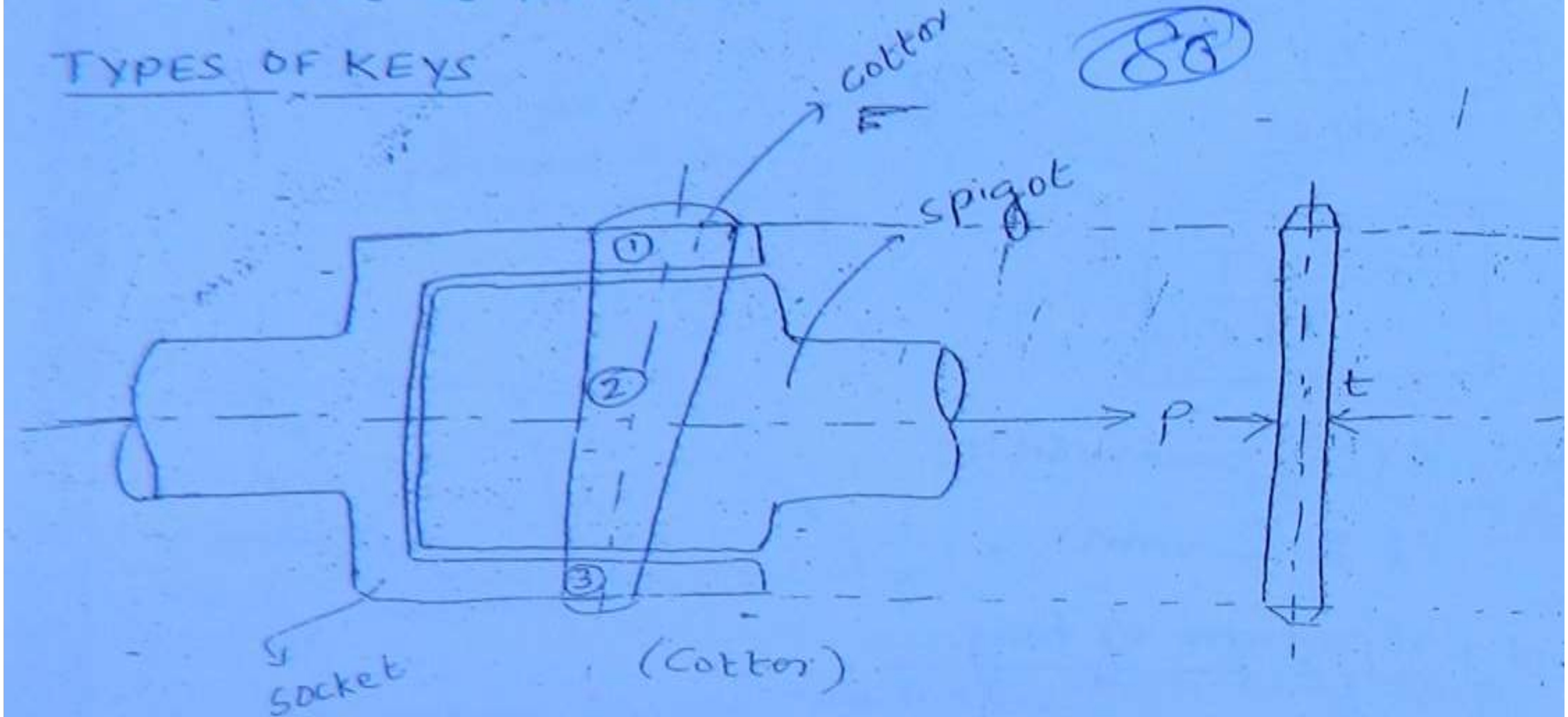
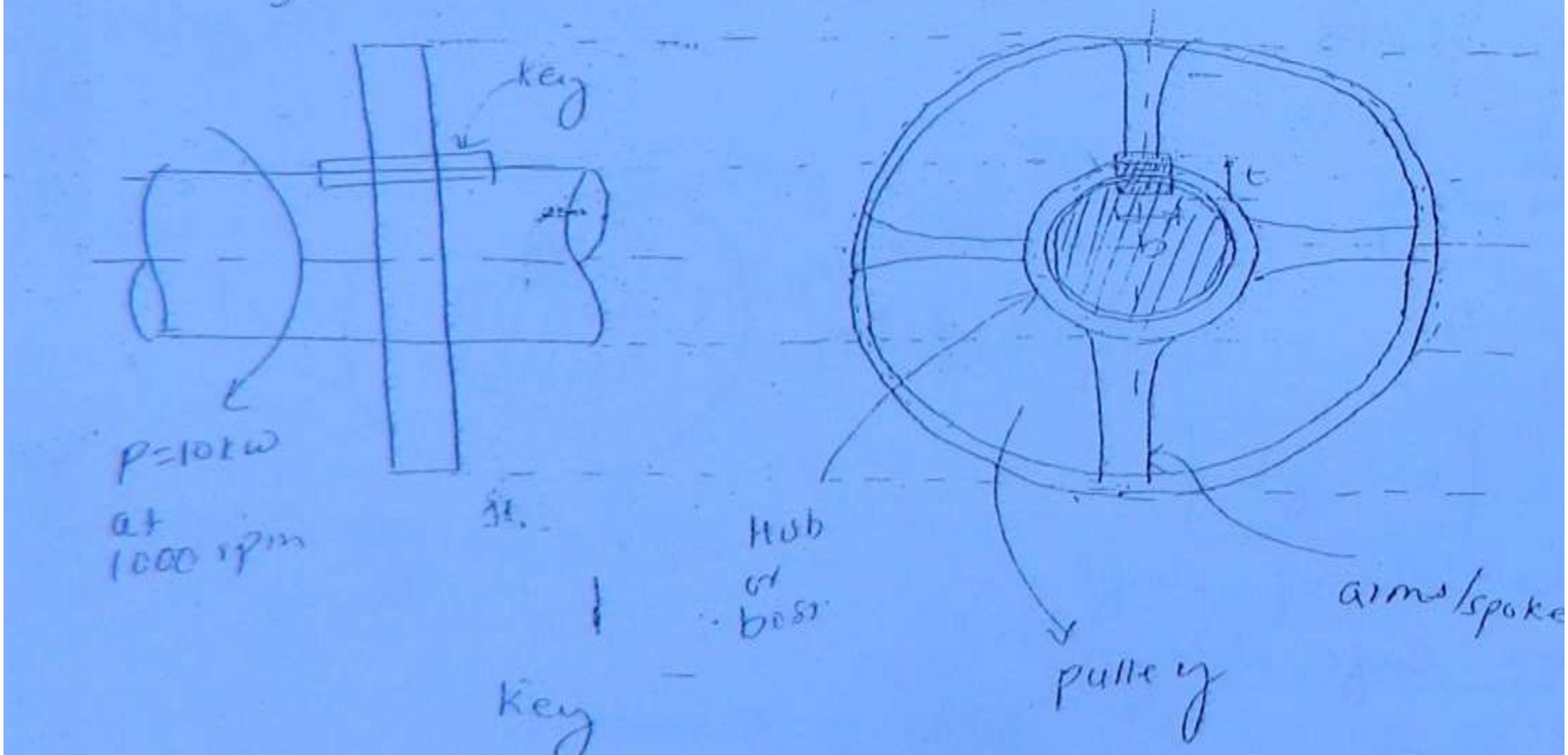
## TYPES OF KEYS



| Key | Cotter |
|---|---|
| 1) Keys are subjected to shear over a longitudnal section | 1) cottors are subjected to shear over a transrerse section |
| (2) parallel to axis of shaft | 2) $\perp$ or to axis of shaft |
| (3) Temperory fastner used for power transmission | 3) used to join two co-axial bar or rods |

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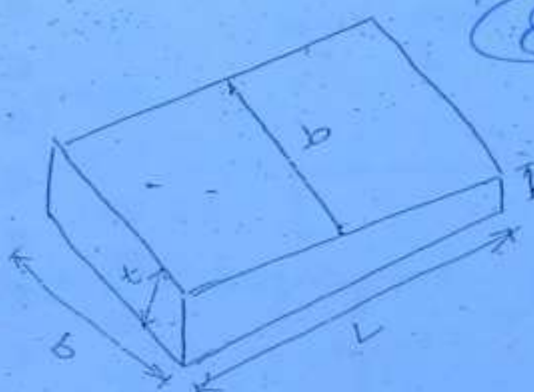
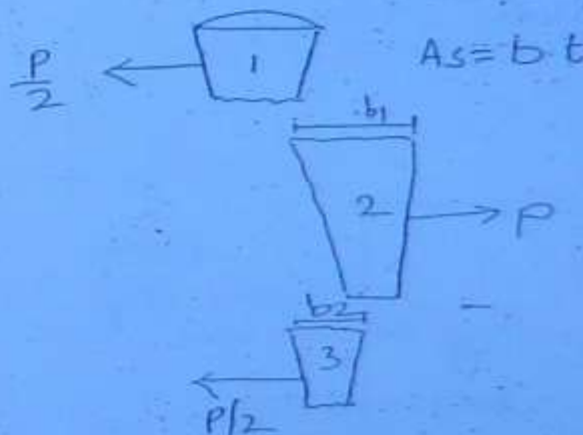



| Key | Cotter |
|---|---|
| (4) They are subjected to single shear | (4) They are subjected to double shear |
| (5) $A_s = L \times b$ | (5) $A_s = b \times t$ |
| (6) Tapper is provided only on Top surface | (6) Tapper is provided on both side |
| (7) Taper is provided on the thickness | (7) Taper is provided on the width |
| (8) $\tau_s = \frac{P_s}{A_s} = \frac{P/2}{b_1 t} = \frac{P}{2 b_1 t}$ | (8) $\tau_s = \frac{P_s}{2A_s} = \frac{P}{(b_1+b_2)t} = \frac{P}{2bt}$ |

$(\tau_{max})_{ind} \leq \tau_{per}$

$\frac{P}{2bt} \leq \tau_{per}$

$\boxed{P \leq 2bt\,\tau_{per}}$

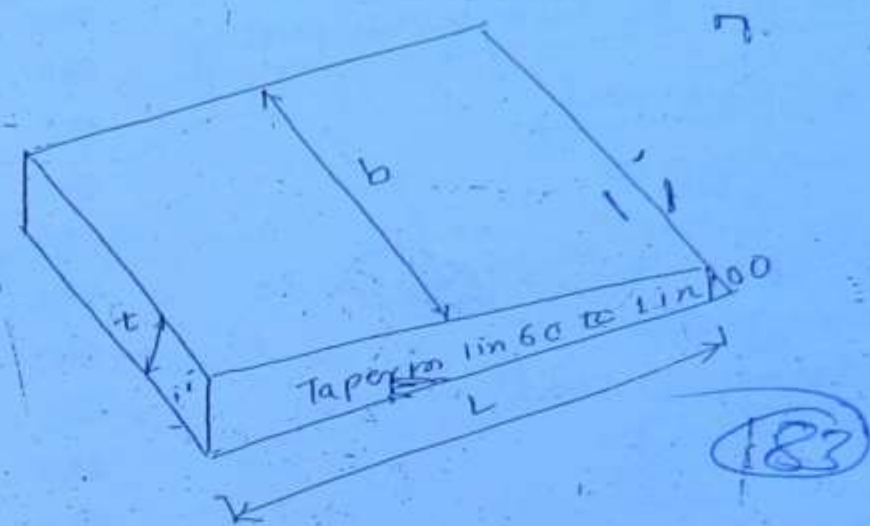
(82)

Shear strength of cotter = $2 \cdot b \cdot t \cdot \tau_{per}$

Types of Keys

- ...w and Medium duty keys
  - ...s which are ...eventing both rotary relative and ...ial Motion
    - ...k ...s
    - SADDLE keys
      - Rectangular or flat key
      - Gib head key
      - woodruff key
      - hollow Saddle Key
      - flat Sadle key
  - Keys which are permitting relative axial Motion
    - feather keys
    - splines (multy keys shaft)
- Heavy duty keys
  - BARTH key
  - kennedy key
  - Tangent keys

7.




Taperkey / Rect sunk key / Flat key


(i) Taper Key (ii) Gib head key

⇒ b = t ⇒ square key

iii) WOODRUFF keys

used in tapered shafts, because of self aligning properties, form of semicircular disk

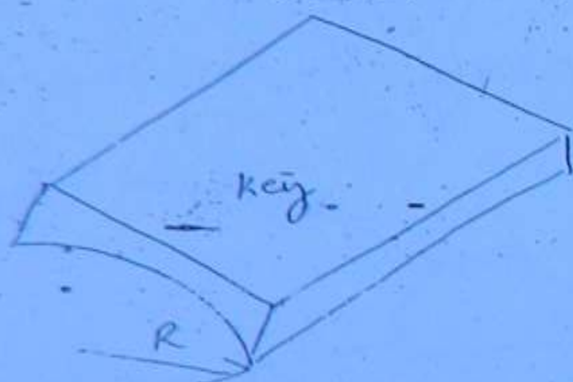
Saddle keys

eyways is present only in the ~~ke~~ hub of the pulley

here only one keyway is required.


(54)

ollow saddle key.

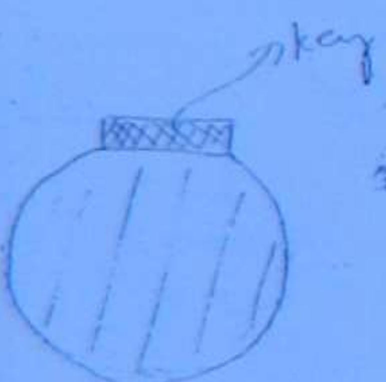



power is transmitted due to frictional forces

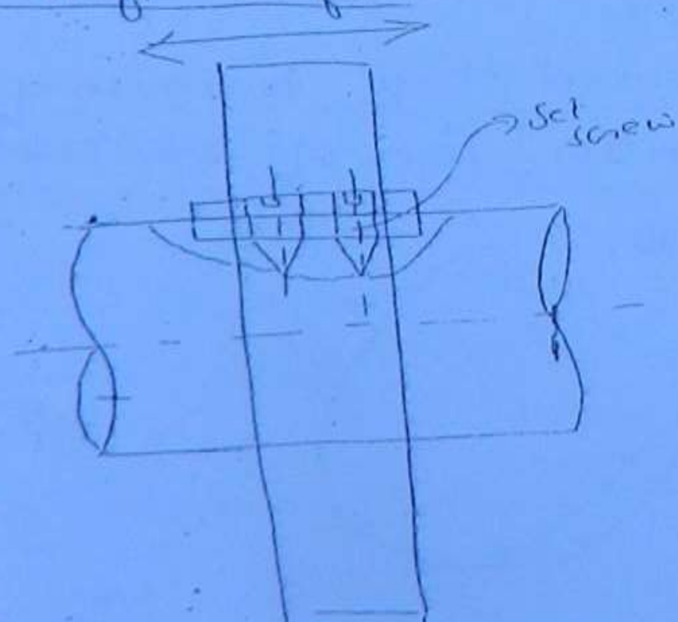
veloped between shaft and key surfaces

d they comes under Low duty key, here key

face is adjusted with shaft surface

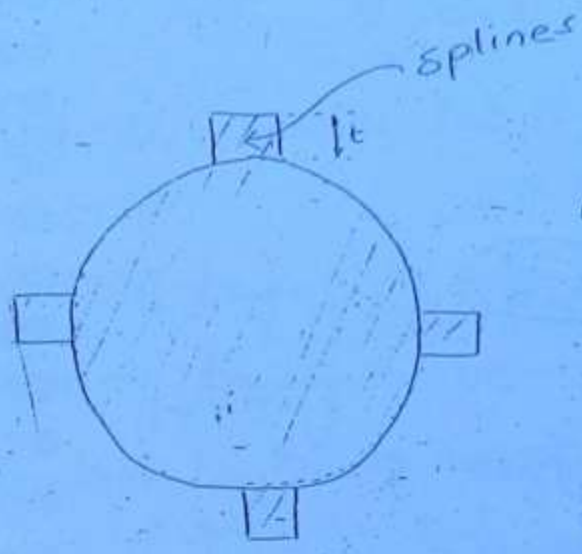



flat saddle key




feather key

## Splines




F.V

a key which is integeral with the shaft are called splines.

~~[illegible]~~

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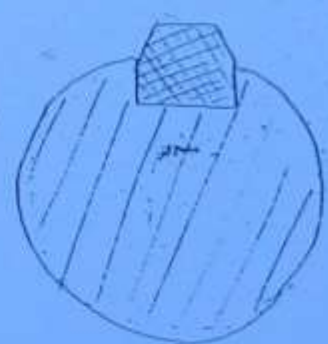
4. Splined shaft (Multi keyed shaft)

## Heavy duty keys

(i) Barth key



it is a modified rectangular key

(ii) Kennedy key


- It is a modified Square key.

here shear area increases and Power transmitting capacity increases.

Tangent Key


(86)

∴ a square of side $\frac{d}{4}$ is to be fitted on a shaft of diameter 'd' and in the hub of the pulley. If the material of key and shaft are same and two are equally strong in shear what is the length of the key :—

(a) $\frac{\pi d}{2}$ (b) $\frac{2\pi d}{3}$ (c) $\frac{3\pi d}{4}$ (d) $\frac{4\pi d}{5}$

Sol<sup></sup>

$$T_s = T_{key}$$

$$\frac{\pi}{16} d^3 \tau_s = \frac{b d L \tau_k}{2}$$

$$\frac{\pi}{16} d^3 = \frac{d}{4} \times \frac{d \times l}{2}$$

$$\therefore L = \frac{\pi d}{2}$$

∴ a square key of side d/4 each and length 'L' is used to transmit torque 't' from the shaft of diameter 'd' to the hub of a pulley. Assuming the length of the key is equal to the thickness of the pulley, the average shear stress developed in the key is given by

4t (b) 16t (c) 8t (d) 16t

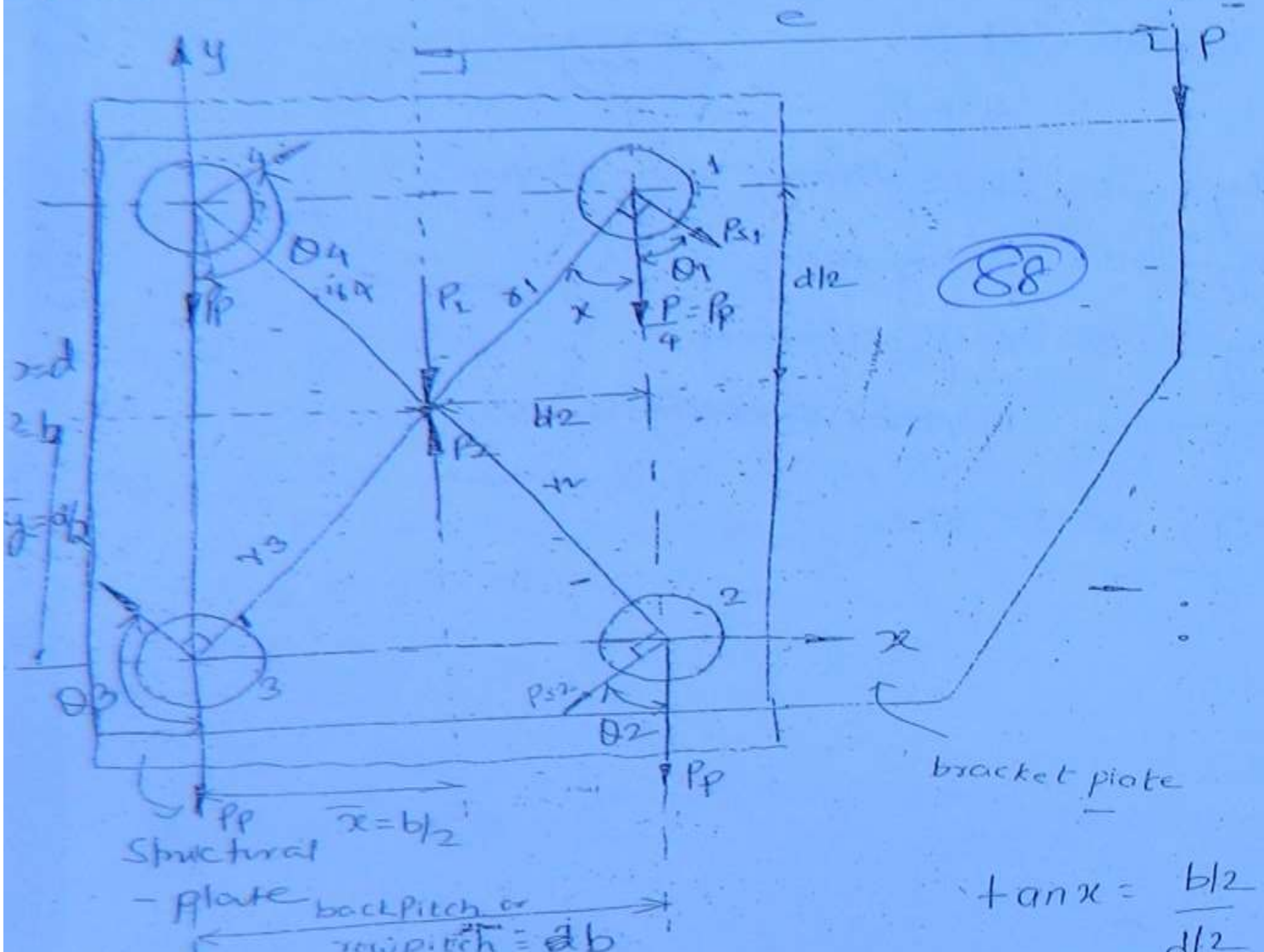
Soln $\tau_s = \frac{2T}{bdl} = \frac{2 \times T}{\frac{d}{4} \times d \times l} = \frac{8T}{ld^2}$

Q: Match list I with list II (87)

| LIST-I | LIST-II |
| --- | --- |
| (a) woodruff key | 1. Loose fitting, light duty |
| (b) Kennedy key | 2. heavy duty |
| (c) feather key | 3. Self aligning |
| (d) flat key | 4. Normal industrial use |

b-2, a-3, c-1, d-4

## DESIGN OF RIVETED JOINT UNDER ECCENTRIC LOADING




$$\tan\alpha = \frac{b/2}{d/2}$$

### determination of C.G of Rivet System

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3 + \cdots}{A_1 + A_2 + A_3 + \cdots}$$

$$= \frac{A[x_1 + x_2 + x_3 + \cdots + x_n]}{n \cdot A}$$

$$\boxed{\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}}$$

$$\bar{x} = \frac{b+b+0+0}{4}$$

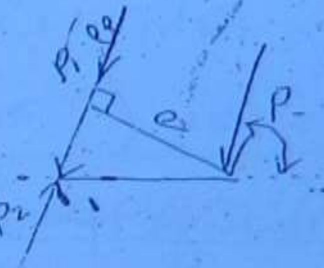
$$\therefore \boxed{\bar{x} = \frac{b}{2}}$$



$$\bar{y} = \frac{y_1+y_2+y_3+y_4}{4} = \frac{d+0+0+d}{4} = \frac{d}{2}$$

(2) Introduce two equal and opposite forces $P_1$ and $P_2$ through centroid Such that $P_1 = P_2 = P$. parallel to the applied load

$$P_1 = P_2 = P$$




(3) e

(4) <u>Effect of $P_1$</u> -

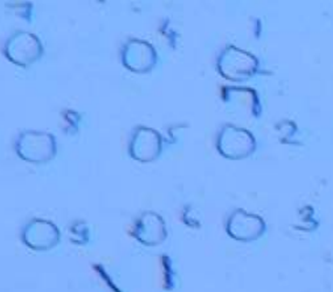
effect of $P_1$ is to cause a primary shear force ($P_P$) of equal magnitude at each and every rivets. $P_P = \frac{P_1}{n} = \frac{P}{4}$

(5) <u>Effect of P and $P_2$</u>

P and $P_2$ causes a twisting couple with respect of group of rivets.

TM$^{t}$ = P. e → clockwise, due to this twisting couple rivets are subjected to a secondary shear force ($P_s$) and the $P_s$ magnitude is directly propotional to 'r' (ie, r = distance between C.G of group of rivets and C.G of each rivets.

$$P_s \propto r$$




$(P_{s1} = P_{s3} = P_{s7} = P_{s9}) > (P_{s2} = P_{s4} = (P_{s6}) = P_{s8}$

90

$P_{s5} = 0.$

hence $P_s$ is maximum at a rivet which is far away from the CG of the rivet system

$P_s$ is at each and every rivet are equal in magnitude when all the rivets are located at same distance from CG of rivet system

$$P_{s1} = P_{s2} = P_{s3} = P_{s4} \; (\because r_1 = r_2 = r_3 = r_4)$$

$P_s$ direction is always perpendicular to the line joining CG of group of rivets and CG of each rivet

Calculation of $r_1, r_2, r_3$ and $r_4$

$$r_1 = r_2 = r_3 = r_4 = \sqrt{\left(\frac{b}{2}\right)^2 + \left(\frac{d}{2}\right)^2}$$

Calculation of $P_{s1}, P_{s2}, P_{s3}$ and $P_{s4}$

$P_{s1} \propto r_1 \Rightarrow P_{s1} = k \cdot r_1$

$P_{s2} \propto r_2 \qquad \therefore k = \frac{P_{s1}}{r_1}$

$P_{s4} \propto r_4 \qquad \Rightarrow P_{s2} = k \cdot r_2$

$$P_{s2} = P_{s1}\left(\frac{r_2}{r_1}\right)$$

$$\Rightarrow P_{s4} = P_{s1}\left[\frac{r_4}{r_1}\right]$$

$$\boxed{P_{sn} = P_{s1}\left[\frac{r_n}{r_1}\right]}$$

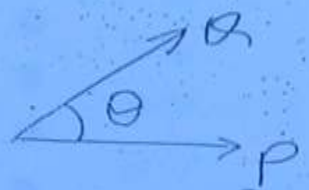
$$P_{s1} \cdot r_1 + P_{s2} \cdot r_2 + P_{s3} \cdot r_3 + P_{s4} \cdot r_4 = P \cdot e$$

$$\boxed{\frac{P_{s1}}{r_1}\left[r_1^2 + r_2^2 + r_3^2 + r_4^2\right] = P \cdot e} \quad (9.1)$$

$\therefore$ $P_{s1}$ can be determined

3). Calculation of $\theta_1, \theta_2, \theta_3, \theta_4$

$(\theta_1 = \theta_2) < (\theta_3 = \theta_4)$




$$\therefore R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$P = P_P, \; Q = P_s$

$(R_1 = R_2) > (R_3 = R_4)$

$R_{max} = R_1$ or $R_2$

hence worst rivets are 1 and 2.

$$R_{max} = R_1 \text{ or } R_2 = \sqrt{P_P^2 + P_{s1}^2 + 2P_P P_{s1} \cos\theta_1}$$

) diameter of rivet

condition for safe design

$(\tau_{max})_{ind} \leq \tau_{per}$

$$\frac{R_{max}}{\frac{\pi}{4}d^2} \leq \tau_{per}$$

shear strength

$$\boxed{R_1 \text{ or } R_2 \leq \frac{\pi}{4} d^2 \tau_{per}} \quad \therefore d \geq ?$$

hear strength of rivet = $k \cdot \frac{\pi}{4} d^2 \tau_s$



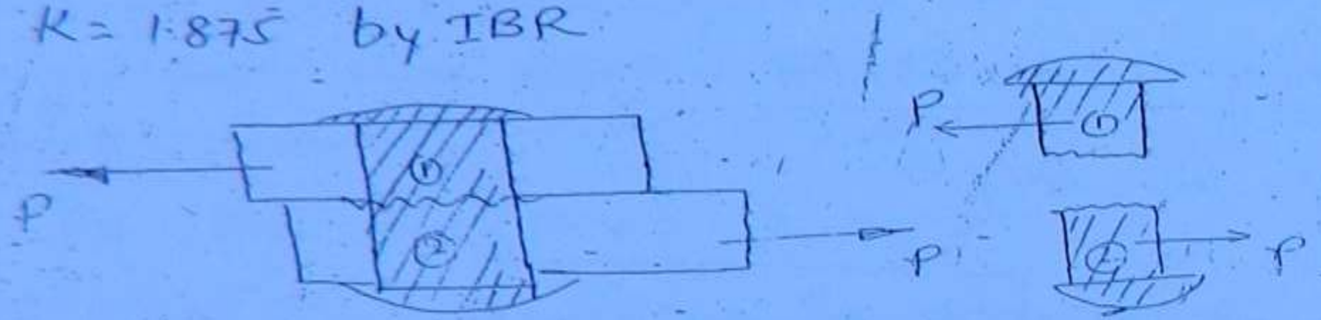
$k = 1 \Rightarrow$ single shear

eg. Lap Joint, single strap butt joint

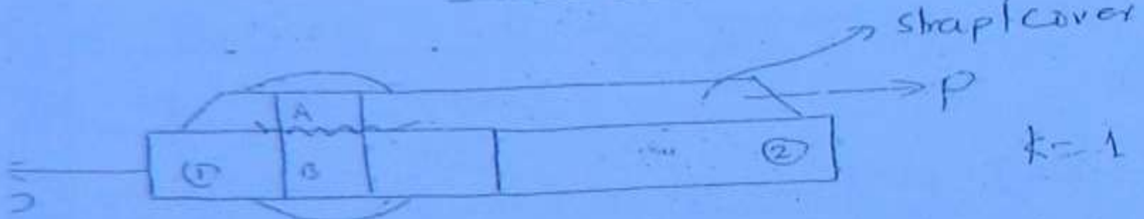
$k = 2 \Rightarrow$ double shear

eg double strap butt joint

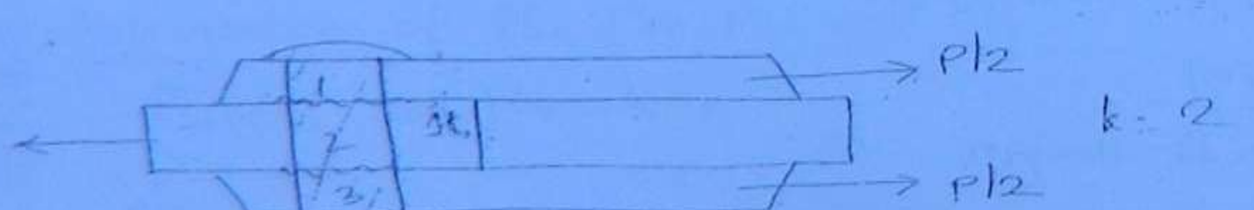
$k = 1.875$ by IBR



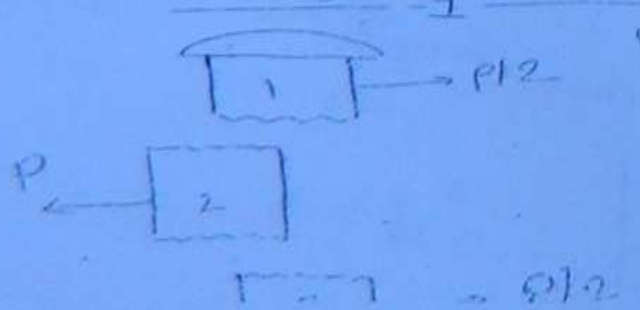

Lap Joint




butt joint (Single strap)

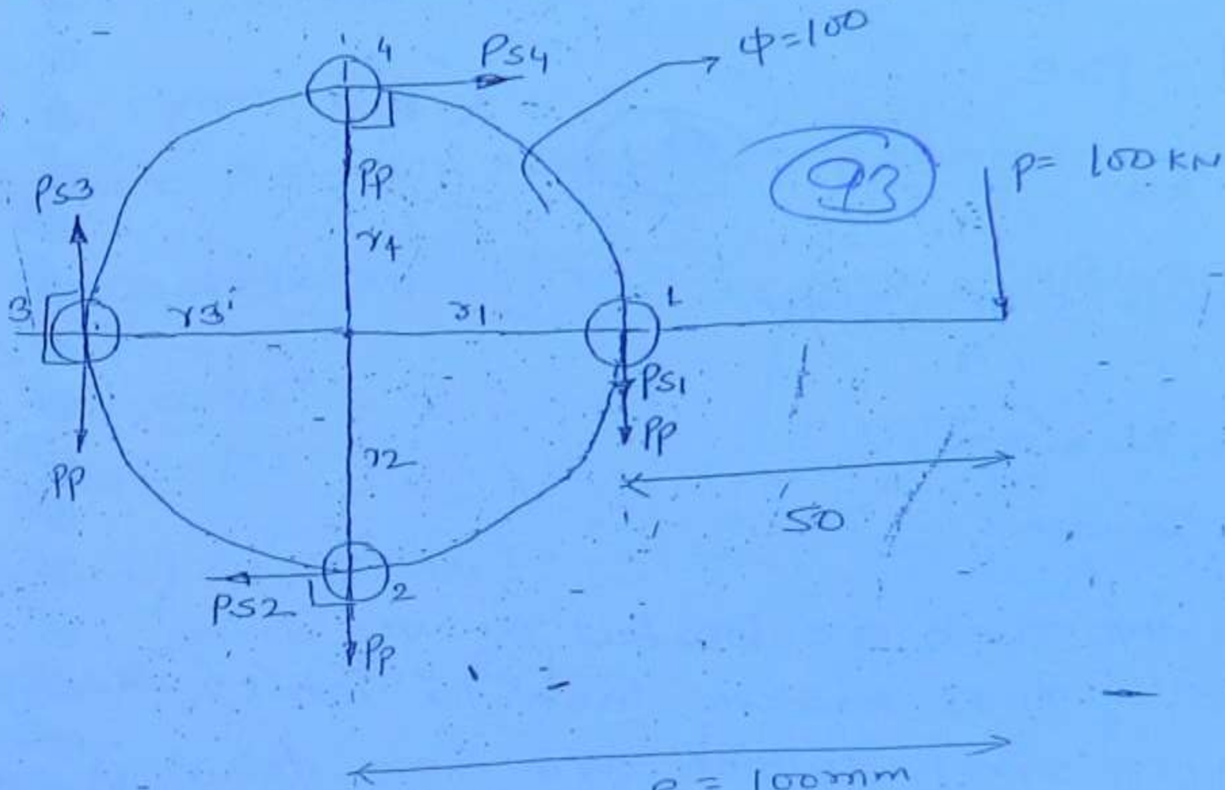



double strap butt joint




Stress is same

Q: design an eccentrically loaded riveted joint as shown in the figure. If the permissible shear stress for the rivet material is 75 mpa.




$P_p = \frac{P_1}{n} = \frac{P}{4} = 25\ kN$

$T_M = P \cdot e = 100 \times 100 = 10^4\ kN \cdot mm = 10^7\ N \cdot mm$

$r_1 = r_2 = r_3 = r_4 = 50\ mm$

$P_{s1} = P_{s2} = P_{s3} = P_{s4} =$

$\theta_1 = 0°, \quad \theta_2 = 90°, \quad \theta_3 = 180°, \quad \theta_4 = 90°$

$\theta_1 < (\theta_2 = \theta_4) < \theta_3$

$R_1 > (R_2 = R_4) > R_3$

$R_{max} = R_1$

$P \longrightarrow, Q \longrightarrow \therefore R = \vec{P} + \vec{Q}$

$R_{max} = P_P + P_{S1} = \frac{\pi}{4} d^2 \tau_s$

$\frac{P_{S1}}{r_1} [4 r_1^2] = P \times e$

$P_{S1} = 50 \text{ kN}$

94

$25 \times 10^3 + 50 \times 10^3 \leq \frac{\pi}{4} d^2 \times 75$

$\therefore d \geq 35.6 \text{ mm}$

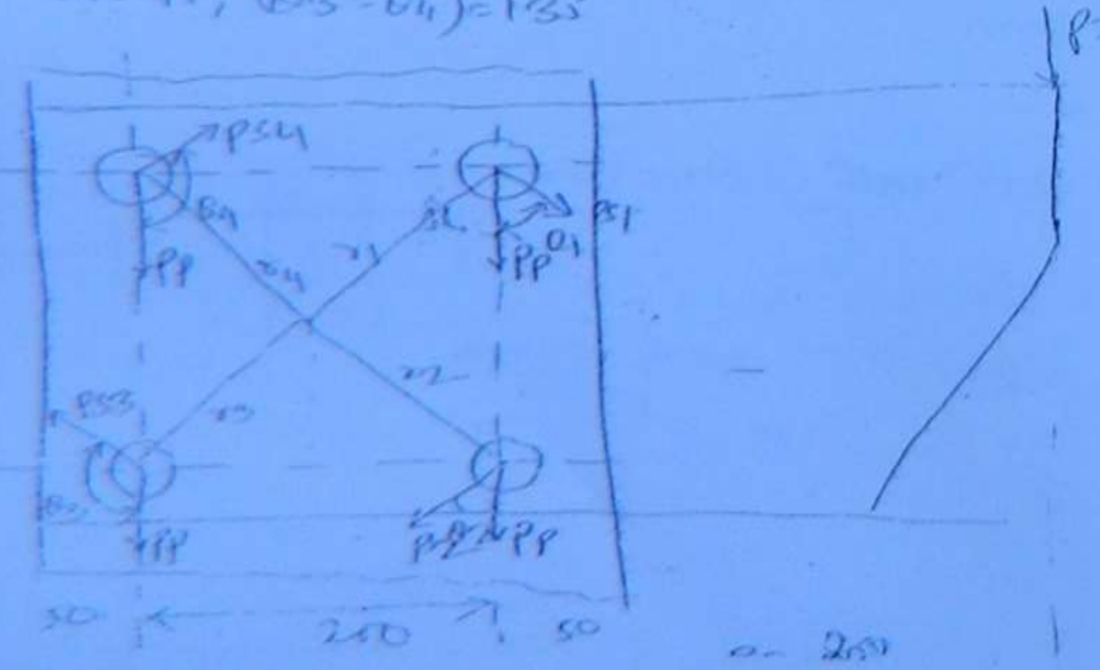
$\therefore d = 36 \text{ mm}$

When all the rivets are located at the same from the C.G of rivet system then the worst rivets are those rivets which are near to the line of action of applied load.

$P = 100$, $b = 200$, $d = 200$, $e = 250$

$\tau_{per} = 60 \text{ MPa}$. $d = ?$ $R_1, R_2, R_3, R_4 = ?$

$\theta_1 = 45$, $(\theta_3 = \theta_4) = 135$



$\bar{x} = 100$

$y = 100$

$P_p = \frac{100}{4} = 25$ KN

$e = 250$ mm

Now $TM = P \cdot e = 100 \times 250 = 25 \times 10^6$ N-mm

Now, $r_1 = r_2 = r_3 = r_4 = \sqrt{(b/2)^2 + (d/2)^2} = 141.42$ mm

$\theta_1 = \theta_2 = 45^\circ$

$\theta_3 = \theta_4 = 135^\circ$

$(\theta_1 = \theta_2) < (\theta_3 = \theta_4)$

$\therefore (R_1 = R_2) > (R_3 = R_4)$

$R_{max} = R_1$ or $R_2$

$R_{max} = \sqrt{P_p^2 + P_{s1}^2 + 2 P_p P_{s1} \cos\theta_1}$

$= \sqrt{(25)^2 + (176.778)^2 + 2 \times 25 \times 176.778 \times \cos 45^\circ}$

$\therefore R_{max} = 195.25$ KN

195

also, $\frac{P_{s1}}{r_1} \times 4r_1^2 = Pe$

$\therefore P_{s1} \times 4r_1 = P \cdot e$

$P_{s1} = \frac{P \cdot e}{r_1} = \frac{100 \times 10^3 \times 250}{141.42}$

$\therefore P_{s1} = 176.778$ KN

$R_{max} = \frac{\pi}{4} d^2 \times \tau_s$

$195.25 = \frac{\pi}{4} d^2 \times 60$
kN

$\Rightarrow 195.25 \times 10^3 = \frac{\pi}{4} d^2 \times 60$

$d = 64.36$

$\boxed{d \simeq 66 \text{ mm}}$

96

Also, $R_3 = R_4$.

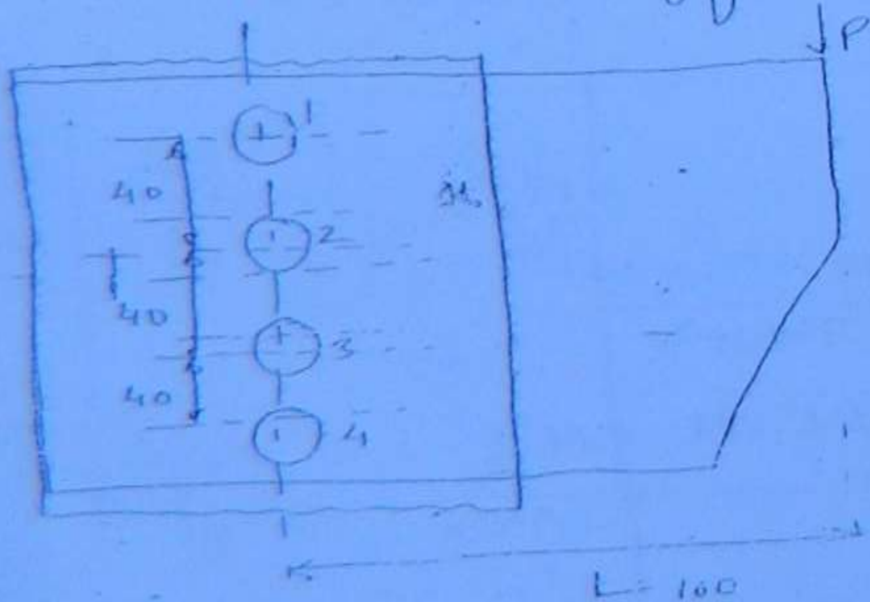
$R_3 = R_4 = \sqrt{P_P^2 + P_{S3}^2 + 2 P_P P_{S3} \cos\theta_3}$

$P_{S1} = P_{S2} = P_{S3} = P_{S4}$

$R_3 = R_4 = \sqrt{(25)^2 + (176.778)^2 + 2 \times 25 \times 176.778 \times \cos 135°}$

$\therefore R_3 = R_4 = 160.079$ kN.

determine which of the following rivets are most loaded rivets in the eccentrically loaded joint as shown in the figure?




$S_{yt} = 280$ MP
$FS = 2$
$d = ?$

(a) All the rivets

(b) 1 only

(c) 4 only

(d) 1 and 4 only

here, $\theta_1 = \theta_2 = \theta_3 = \theta_4 = 90°$

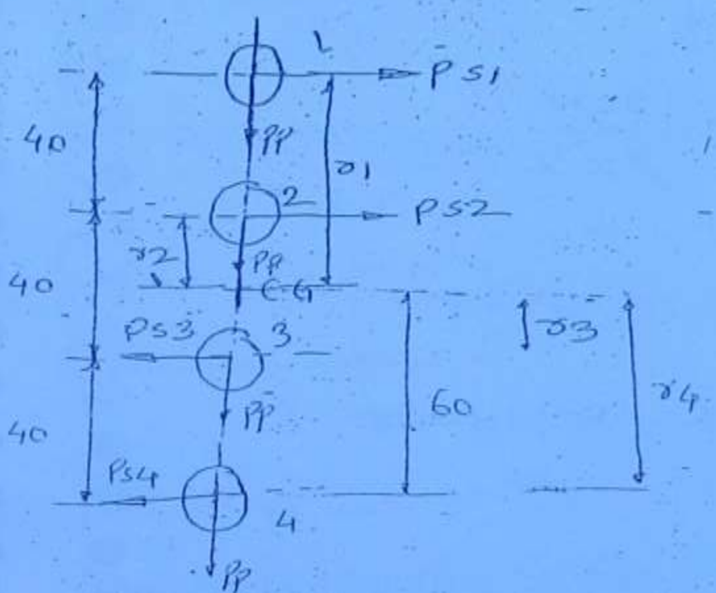
$(P_{s1} = P_{s4}) > (P_{s3} = P_{s2})$

$(r_1 = r_4) > (r_3 = r_2)$

$\therefore (R_1 = R_4) > (R_2 = R_3)$

$\therefore R_{max} = R_1$ or $R_4$

(97)




when all the rivets are arranged in a single vertical row the worst rivets are those rivets which are far away from the C.G of group of rivets

$$R_{max} = R_1 \text{ or } R_4 = \sqrt{P_p^2 + P_{s1}^2} = \frac{\pi}{4} d^2 \tau_s$$

$$= \frac{\pi}{4} d^2 \times \frac{S_{ys}}{N} \text{ @ } \frac{S_{yt}}{2N}$$

$$P_p = \frac{P}{4} = 12.5 \text{ kN}$$

$r_1 = r_4 = 60$, $r_2 = r_3 = 20$ mm

$$\frac{P_{s1}}{r_1}\left[2r_1^2+2r_2^2\right] = P \cdot e$$

$$\frac{P_{s1}}{60}\left[2\times 60^2+2\times 20^2\right] = 50\times 100$$

$$P_{s1} = 37.5 \text{ kN}$$

3) $R_{max} = P_{s1}$ or $P_{s4}$ (98)

$$R_{max} = \sqrt{P_p^2+P_{s1}^2} = \frac{\pi}{4} d^2 \frac{S_{yt}}{2N}$$

$$\Rightarrow \sqrt{12.5\times 10^3+(37.5)^2} = \frac{\pi}{4} d^2 \times \frac{200}{2\times 2}$$

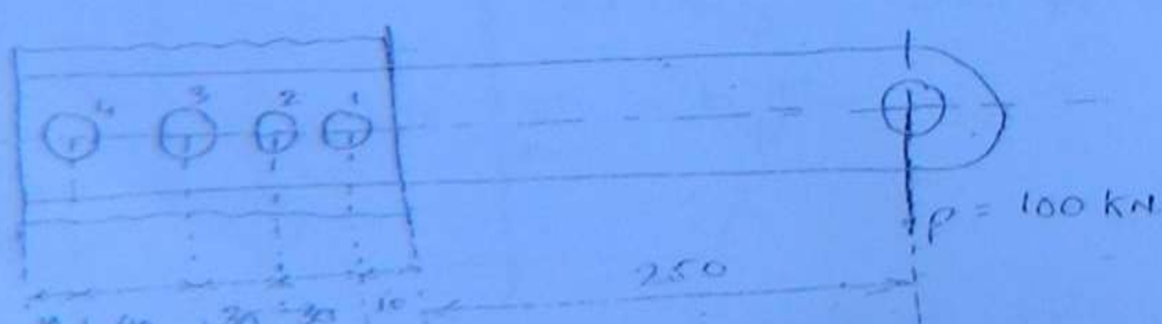
$\therefore d = 31.7$

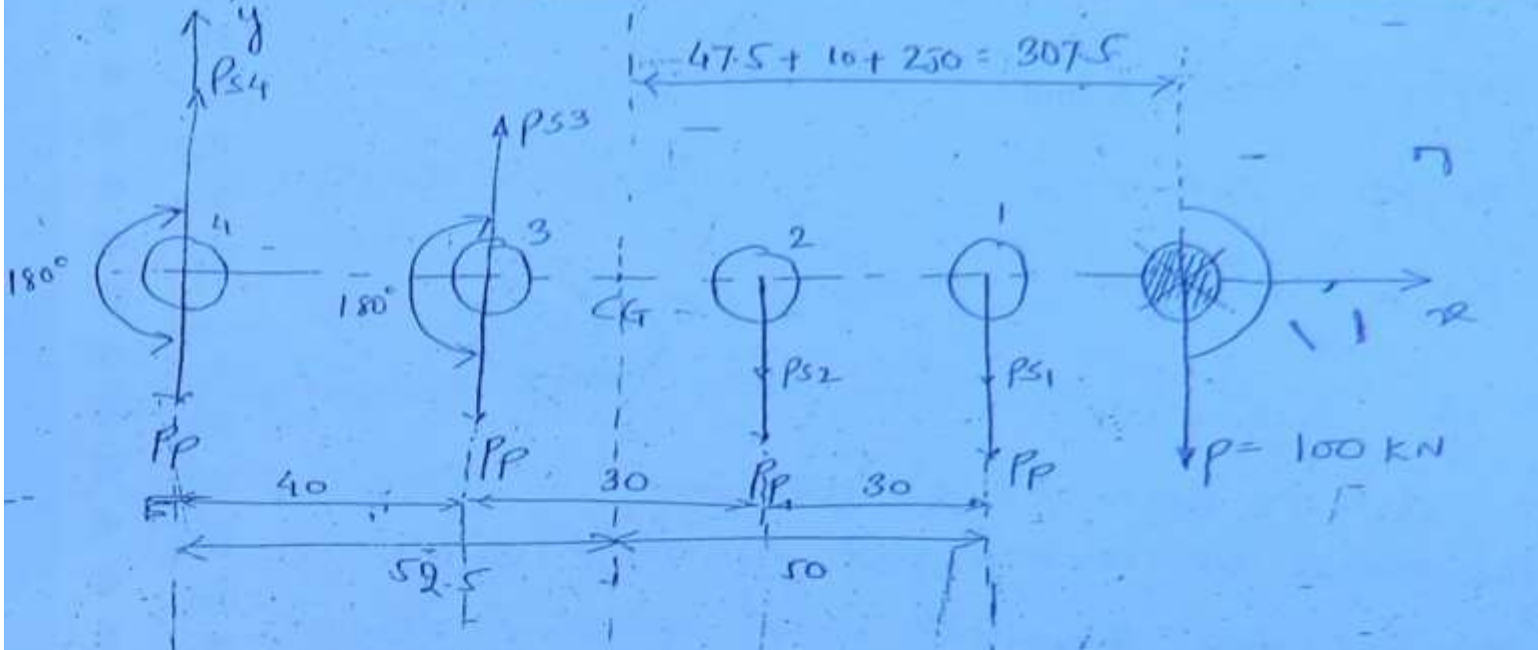
$\therefore d \simeq 32$ mm

Repeat the above question for the resultant force in all the rivets?

$$P_{s2} = P_{s1}\left(\frac{r_2}{r_1}\right)$$

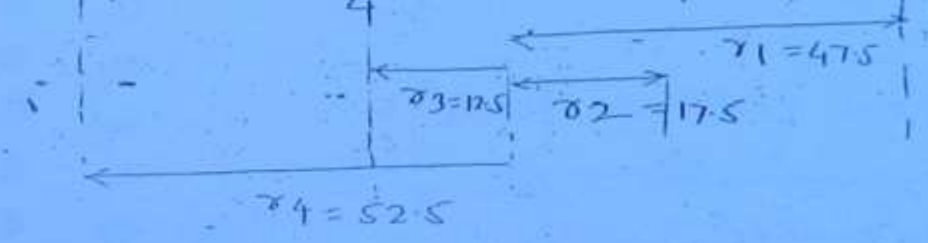
For eccentrically loaded riveted joint as shown in the figure which of the following rivets is worst load rivet and also determine diameter of the rivets?






$$\bar{x} = \frac{100 + 70 + 40 + 0}{4} = \frac{210}{4} = 52.5$$

99




$r_4 > r_1 > r_2 > r_3$

$\therefore \; P_{s4} > P_{s1} > P_{s2} > P_{s3}$

$(P_s)_{max} = P_{s4}$

$\theta_1 = \theta_2 = 0$

$\theta_3 = \theta_4 = 180°$

$R_1 = P_p + P_{s1}$

$R_4 = P_{s4} - P_p$

$P_p = \frac{100}{4} = 25 \text{ kN}$

$$\frac{P_{s1}}{r_1}\left[r_1^2 + r_2^2 + r_3^2 + r_4^2\right] = P \cdot e$$

$\therefore \; P_{s1} = 266.78 \text{ kN}$

$P_{s4} = P_{s1}\left[\frac{r_4}{r_1}\right] = 294.8 \text{ kN}$

$R_1 = P_p + P_{s1} = 293.78\ kN$

$R_4 = P_{s4} - P_p = 269.8\ kN$

$R_{max} = R_1 = \frac{\pi}{4} d^2 \cdot \tau_s$

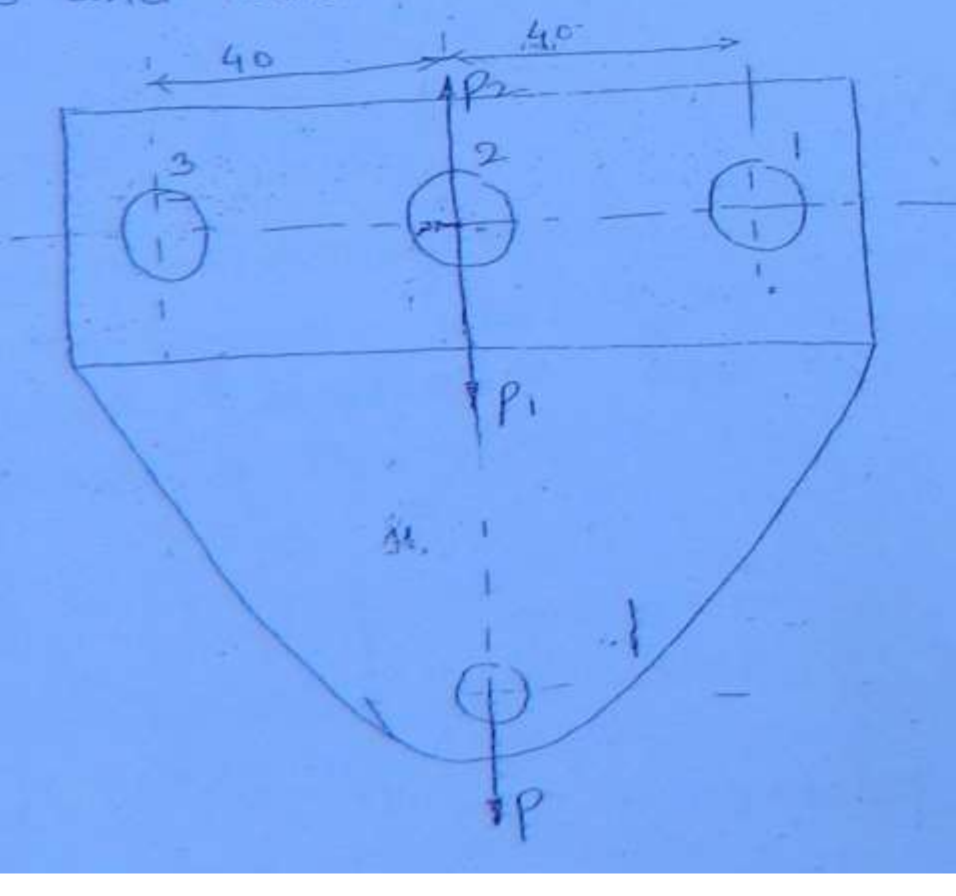
(10)

$293.78 = \frac{\pi}{4} d^2 \times 60$

$d_i = 78.68 \quad \therefore d = 80\ mm$

If all the rivets are arranged symmetrically a single horizontal row then the worst rivet is the rivet which is near to line of action of the load

for an ~~eccentrically~~ Loaded riveted Joint as shown in the fig determine diameter of the rivets and worst loaded rivets?



$TM = 0 \longrightarrow e = 0$

$\therefore P_{s1} = P_{s2} = P_{s3} = 0$

$\therefore P_p = \dfrac{P}{3}$

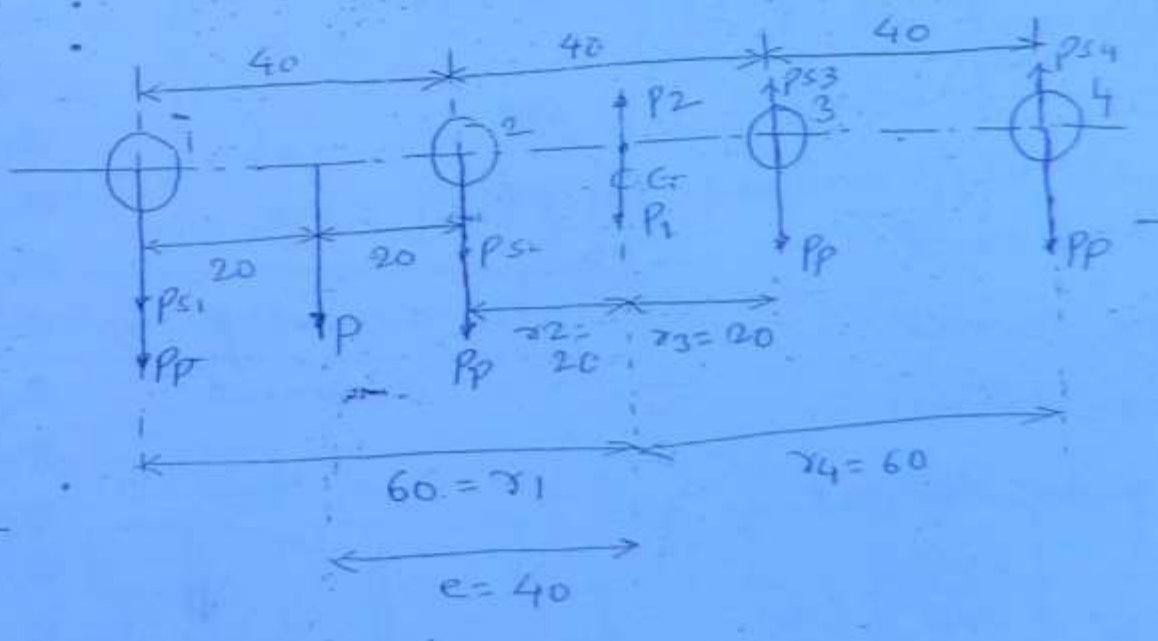
$(\tau_{max})_{md} \leq \tau_{per}$

$\therefore \dfrac{P_s}{A_s} \leq \tau_s \Rightarrow \dfrac{P}{3 \times \frac{\pi}{4} d^2} \leq \tau_s$

$\therefore d \geq \sqrt{\dfrac{4P}{3\pi \tau_s}}$

(10)

Q




$(P_s)_{max} = P_{s1}$ or $P_{s4}$

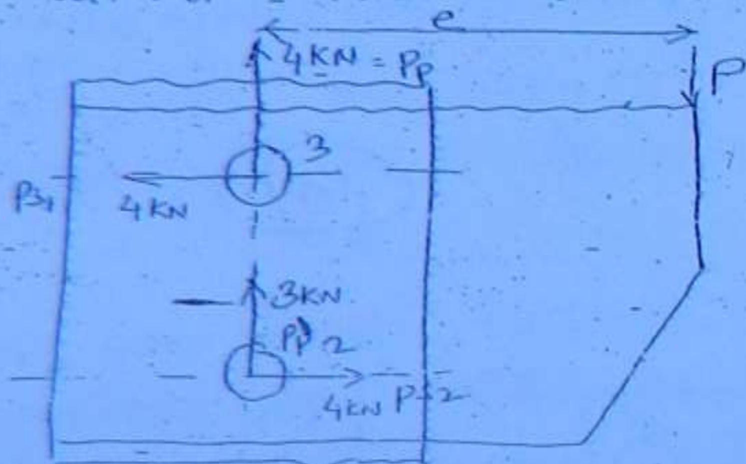
$\theta_{min} = \theta_1$ or $\theta_2$

$R_{max} = R_1 - P_p + P_{s1} = \dfrac{\pi}{4} d^2 \tau_s$

for an eccentrically loaded riveted joint as shown in the fig. which of the following statements are correct if area of rivets is 100 mm2

7.

- eccentricity of the load is 100 mm
- The maximum shear stress in all the rivets 50 MPa
- The total load applied on the joint 6 kN
- resultant force in all the rivets is 5 kN.




(102)

(a) 1, 3, 4 are correct

(b) 1, 2, 4 are correct

(c) 2, 3, 4 are correct

(d) 2 and 4 are correct

$$\frac{P_{s1}}{r_1}[2r_1^2] = P \cdot e$$

$\therefore$ e = cannot be determined.

$$P = n \cdot P_p = 2 \times 3 = 6 \text{ kN}$$

$$R_1 = R_2 = \sqrt{3^2 + 4^2} = 5 \text{ kN}$$

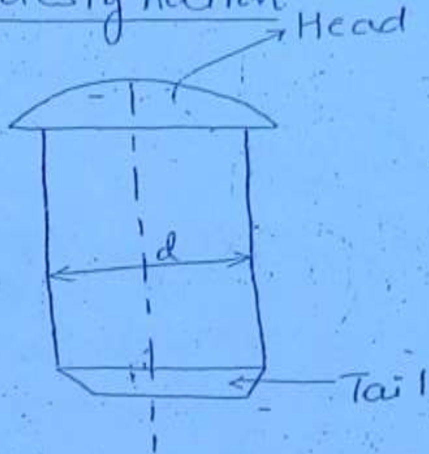
$$R_{max} = R_1 \text{ or } R_2$$

$$\tau_{max} = \frac{R_1 \text{ or } R_2}{A} = \frac{5000}{100} = 50 \text{ mpa}$$

# 6) ECCENTRICALLY LOADED BOLTED JOINT

a) Rivet designation

(Upset forging is used in making bolt heads)




Rivet is specified by its
(i) diameter of shank
(ii) Type of head

(i) snap head rivet

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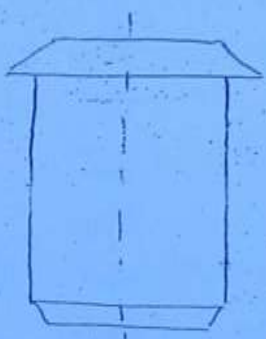
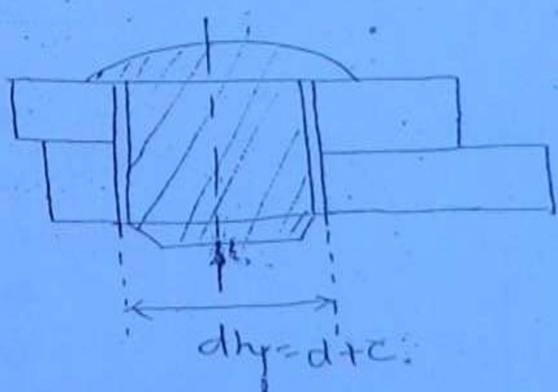


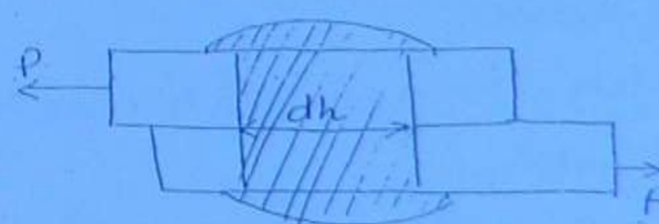
empirical formula for dia of rivet is $\boxed{d = 6\sqrt{t}}$

$t \geq 8\,mm$

above is Unwin's formula

$\boxed{\text{diameter of hole} = d + c}$

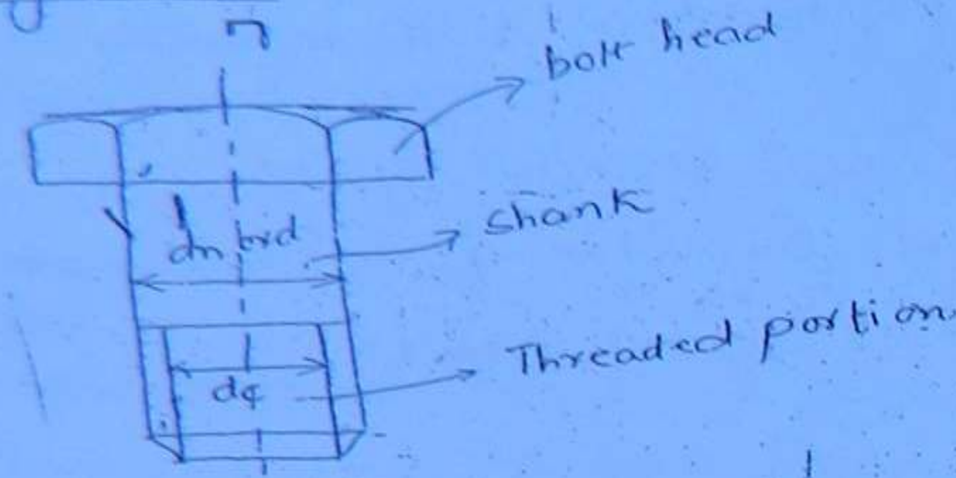



$P_t = (p - d_h)\, t\, \sigma_t$

$P_s = K \cdot \frac{\pi}{4} d^2 \cdot \tau_s$

during the calculation of strength of the rivet shank or rivet diameter is taken into consideration olt designation




04

dc = core diameter

dn = nominal diameter or major diameter

⇒ M 20 ← nominal diameter

↑

thread profile ⇒ M 20 × 2 ← pitch or fine pitch

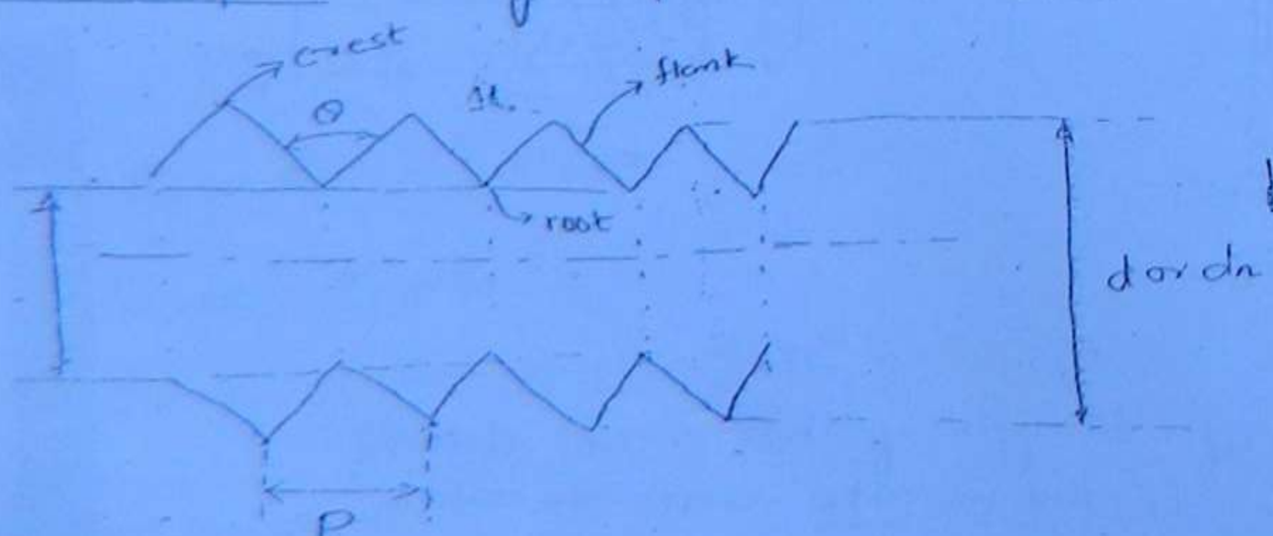
No pitch indicated means = $\boxed{\text{Coarse pitch}}$

⇒ SQ 20 × 2 (for power transmission)

↳ square thread

⇒ ACME 20×2

Coarse pitch = largest pitch available



Major or Nominal diameter:- it is defined as the diameter of an imaginary circle passing through the crest of external thread or roots of an internal thread. (105)

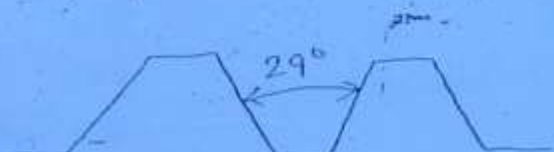
Minor diameter or Core diameter - of a thread is defined as the diameter of an imaginary circle passing through the roots of an external thread or crest of an internal thread.

Thread angle: The angle between the adjacent flank ie, ~~included~~ inclined angle

— $L = n \cdot p$ , L→ Lead

(1)  → Square thread

⇒ in square thread, $\theta = 0°$

(2) 

→ ACME thread

} Power Transmission in 2 direction

⇒ in acme thread $\theta = 29°$

(3) 

→ Buttress thread

⇒ angle in Buttress thread = 45°

} Transmit power in one direction eg presses

→ fine pitch

| itches / dia | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| 20 | ✓ | ✓ | ✓ | ✗ | ✓ |
| 24 | ✓ | ✗ | ✗ | ✓ | ✗ |

→ Coarse pitch

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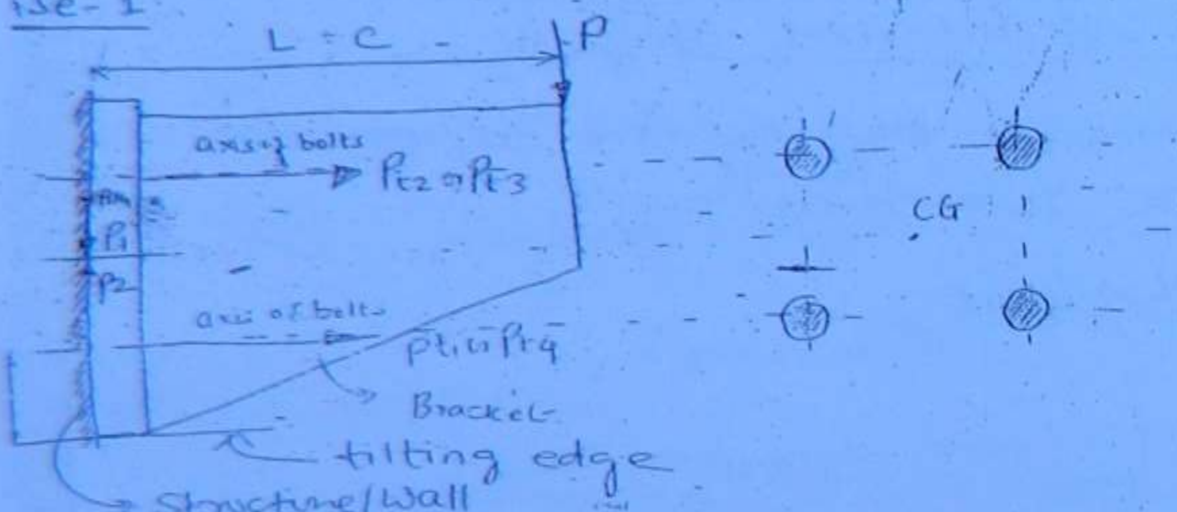
ase- 1




ad is acting in a plane ||el to plane of bolts

Load is acting ⊥ar to axis of bolts

bolts are subjected to shear and tensile stress.

Introduce $P_1 = P_2 = P$

$e = L$

<u>effect of $P_1$</u>

is to cause a shear force of equal magnitude at each and every bolt

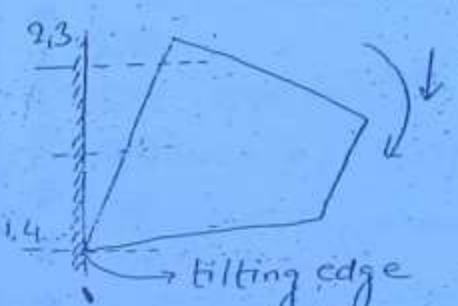
$$P_{shear} = \frac{P_1}{n} = \frac{P}{n} = \frac{P}{4} = P_s \text{ [direct shear]}$$

$$\boxed{\tau_s = \frac{P_s}{\frac{\pi}{4}d_c^2} = \frac{4P_s}{\pi d_c^2}}$$

$$\therefore \tau_s = \frac{x}{d_c^2} \text{ Mpa} \quad -(1)$$

4] Effect of $P$ & $P_2'$

∴ Couple $= P_i e = P \cdot L$




due to this couple as the bracket bends bolts are elongated due to a $\boxed{\text{tensile force}}$

The elongation of the bolts is max which are ~~for~~ away from the tilting edge.

$$\boxed{P_t \propto l}$$

and the tensile force in the bolts is directly proportional to $l$

$$(P_{t2} = P_{t3}) > (P_{t1} = P_{t4})$$

because $(l_2 = l_3) > (l_1 = l_4)$

$$\therefore (P_t)_{max} = P_{t2} \text{ or } P_{t3}$$

∴ 2 and 3 are worst bolts

$$(P_t)_n = P_{t1}\left(\frac{l_n}{l_1}\right)$$

5] Calculation of $(P_t)_{max}$

$$P_{t1}l_1 + P_{t2}l_2 + \cdots = P \cdot e$$

$$\frac{P_{t1}}{l_1}\left[l_1^2 + l_2^2 + \cdots + \right] = P \cdot e$$

$$\therefore P_{t1} = ?$$

$$\therefore (P_t)_{max} = P_{t1} \text{ or } P_{t2} = P_{t1}\left(\frac{l_2 \text{ or } l_3}{l_1}\right)$$

6) $$(\sigma_t)_{max} = \frac{P_{tmax}}{\frac{\pi}{4}d_c^2} = \frac{4(P_t)_{max}}{\pi d_c^2}$$

$$= \frac{y}{d_c^2} \text{ Mpa} \quad \rightarrow (2)$$

(7) diameter of bolts ($d_c$ or $d_n$)

here bolts are designed by using either MSST or MDET because they are subjected to combined stress and made up of ductile mater-

MSST

$$\sigma_t = \frac{S_{yt}}{N} = \sqrt{\sigma_x^2 + 4\tau_{xy}^2}$$

$$\tau_s = \frac{S_{ys}}{N} = \frac{1}{2}\left(\frac{S_{yt}}{N}\right) = \frac{1}{2}\sqrt{(\sigma_{tmax})^2 + 4\tau_s^2}$$

$$\tau_s = \frac{1}{2}\sqrt{\left(\frac{y}{d_c}\right)^2 + 4\left(\frac{x}{d_c^2}\right)^2}, \quad d_c = ?$$

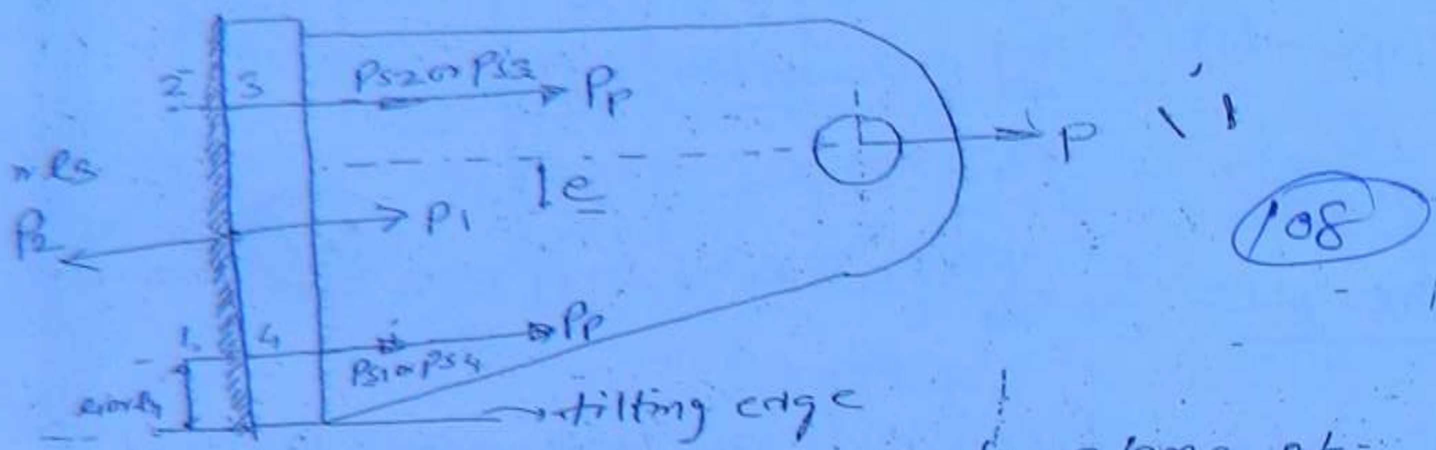
MDET

$$\sigma_t = \frac{S_{yt}}{N} = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

$$\sigma_t = \frac{S_{yt}}{N} = \sqrt{\left(\frac{y}{d_c}\right)^2 + \left(3\left(\frac{x}{d_c}\right)^2\right)}$$

$$d_c = ?$$

Case-2.




Load is acting in a plane ⊥or to plane of bolts / ‖el to Axis of bolts

bolts are subjected to primary tensile and secondary

Note

determination of C.G of group of bolts

Introduce two equal and opposite force parallel to P

$P_1 = P_2 = P$

$e = ?$

effect of $P_1$

is to cause a primary tensile force of same magnitude at each and every bolt

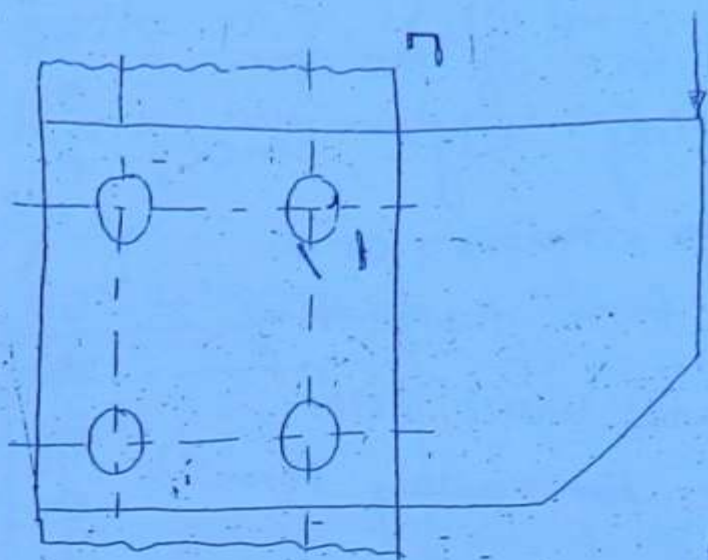
$$P_P = \frac{P_1}{n} = \frac{P_1}{4} = \frac{P}{4}$$

Effect of P and $P_2$

causes a couple $= P \times e$

due to this couple as bracket bends bolts are elongated due to a secondary tensile force.

Case-3

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Load is acting in the plane but away from the plane of bolts.

⇒ subjected to primary and secondary shear stress

$$R_{max} = \frac{\pi}{4} d_c^2 \tau_s$$

$$d_c = ?$$

$$\boxed{d_n = \frac{d_c}{0.84}}$$

econdary tensile force ($P_s$) magnitude is
irectly propotional to distance of the axis
f the bolt from the tilting edge, hence
econdary tensile force is maximum at a
olt which is far away from the tilting
dge, hence in this case worst bolts
re those bolts which are far away from
he tilting edge.

$$P_s \propto l$$

$$(l_2 = l_3) > (l_1 = l_4)$$

$$(P_{s2} = P_{s3}) > (P_{s1} = P_{s4})$$

$$(P_s)_{max} = P_{s2} \text{ or } P_{s3} = P_{s1}\left(\frac{l_2 \text{ or } l_3}{l_1}\right)$$

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$$\frac{P_{s1}}{l_1}\left[l_1^2 + l_2^2 + l_3^2 + l_4^2\right] = P \cdot e$$

$$P_{s1} = ?$$

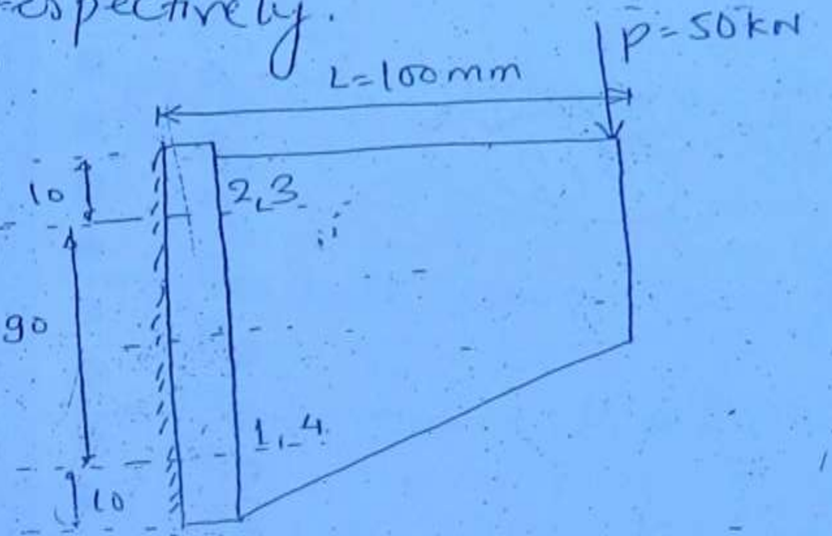
$$(P_s)_{max} = ?$$

$$R_{max} = R_2 \text{ or } R_3 = P_p + P_{s2} \text{ or } P_{s3} = \frac{\pi}{4} d_c^2 \sigma_t$$

$$\therefore d_c = ?$$

$$\therefore d_n = \frac{d_c}{0.84}$$

Q.1 design an eccentrically loaded bolted joint as shown in the fig if ($\sigma_t$)per, tensile and shear stress of bolt material are 100 mpa and 60 Mpa respectively.

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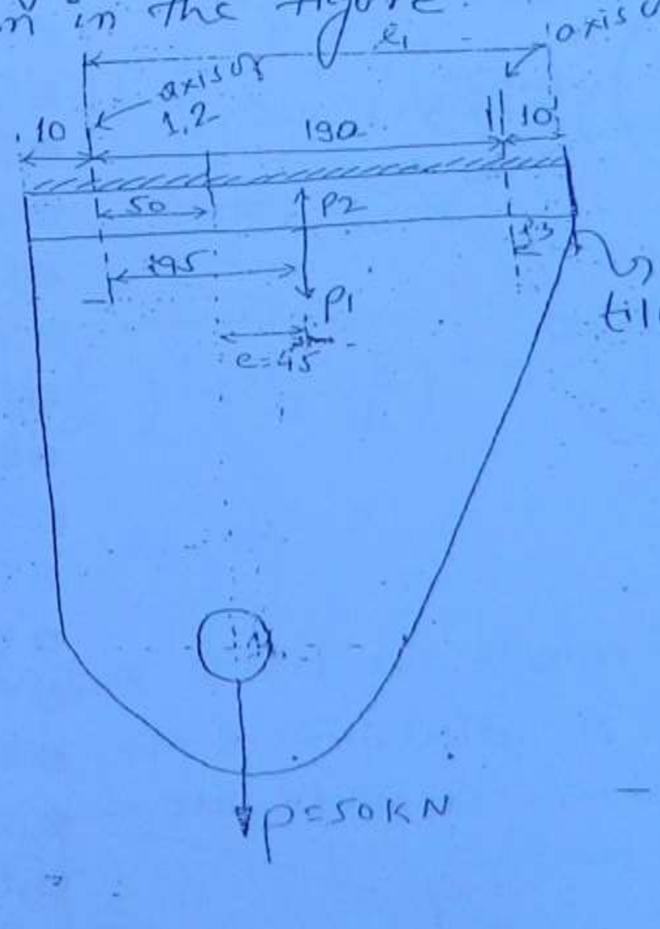
Q.2 Design an eccentrically loaded bolted joint as shown in the figure.




take $\sigma_t$ = 100 mpa

$\tau_s$ = 60 Mpa

$R_{max} = R_1$ or $R_2 = P_p + P_{s1}$ or $P_{s2} = \frac{\pi}{4} d_c^2 \cdot \sigma_t$

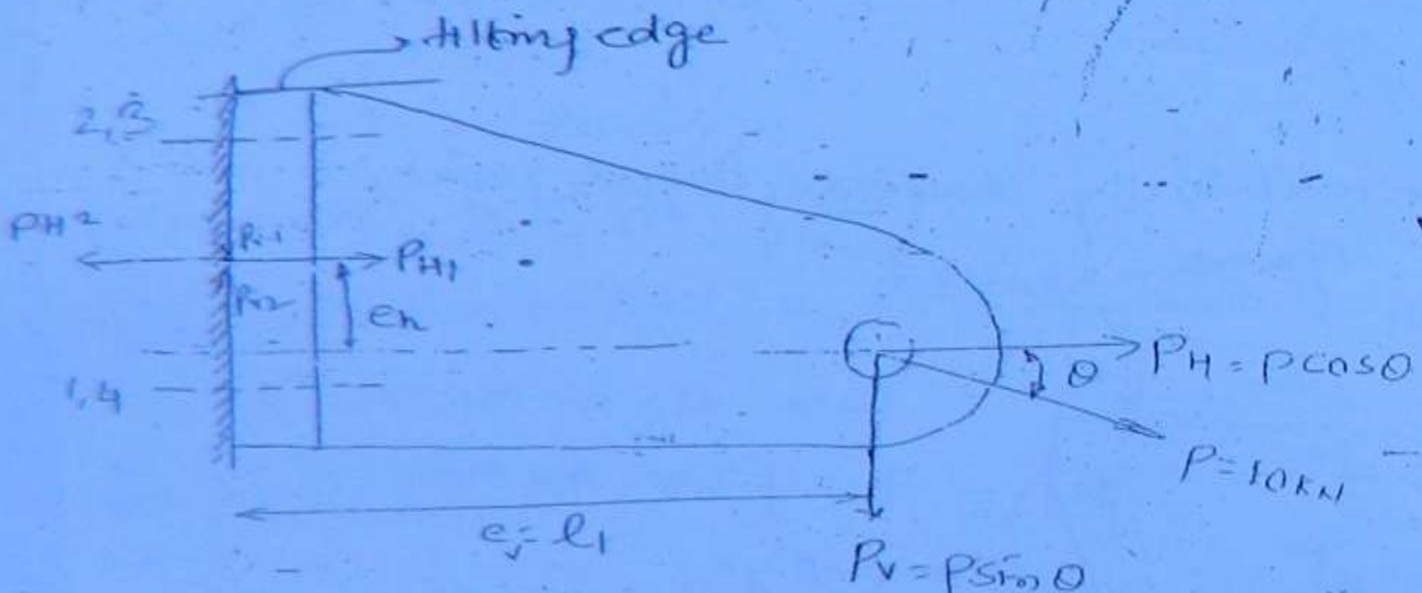
$P_p = \frac{P}{4} = 12.5 \times 10^3\ N$

$\frac{P_{s1}}{l_1}\left[2l_1^2 + 2l_3^2\right] = P \cdot e$

1/12

$\therefore P_{s1} = ?$

$\therefore d_c = ? \quad \therefore d_n = \frac{d_c}{0.84}$




$P_H$ [-ve tensile force]

$M_V = P_V \times l_1 \cdot (CW)$

$M_H = P_H \cdot e_H\ (ACW)$

$M_R = M_H - M_V \quad (ACW)$

$P_s = \frac{P_V}{n}$ [effect of $P_{V1}$] –

$\tau_s = \frac{4P_s}{\pi d_{c}^2} = \frac{x}{d_c^2} \rightarrow (1)\ MPa$

(assuming $M_H$ is more)

effect of $P_H$

$$P_p = \frac{P_H}{4}$$

(113)

effect of $M_R$ :-

$$P_{s1} = P_{s4} = (P_s)_{max} = \frac{P_{s1}}{e_1}\left[2e_1^2 + 2e_2^2\right] = M_R$$

$\therefore P_{s1} = ?$

It involves both case 1 and case-?)

Resultant tensile max $= R_1$ or $R_4 = P_p + P_{s1}$

$$\sigma_t)_{max} = \frac{4R_{max}}{\pi d_c^2} = \frac{Y}{d_c^2} \text{ --- (2)}$$

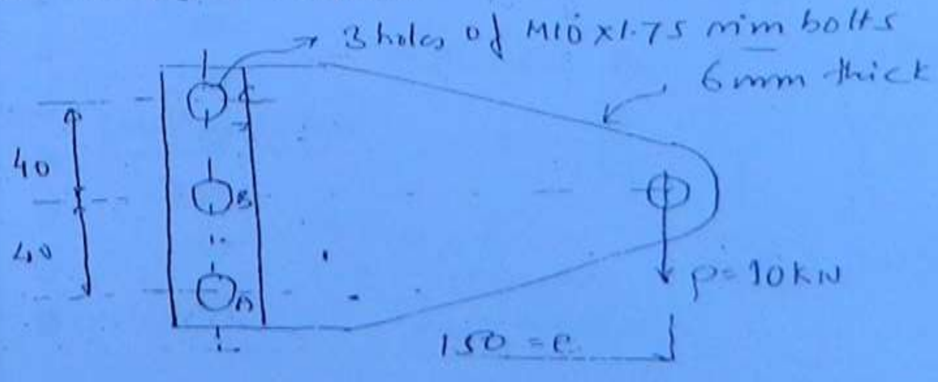
$$\tau_s = \frac{1}{2}\sqrt{\left(\frac{Y}{d_c}\right)^2 + 4\left(\frac{X}{d_c}\right)^2} \quad \text{MSST}$$

$\therefore d_c = ?$....

$$\sigma_t = \sqrt{\left(\frac{Y}{d_c}\right)^2 + 3\left(\frac{X}{d_c}\right)^2} \quad \text{MDET}$$

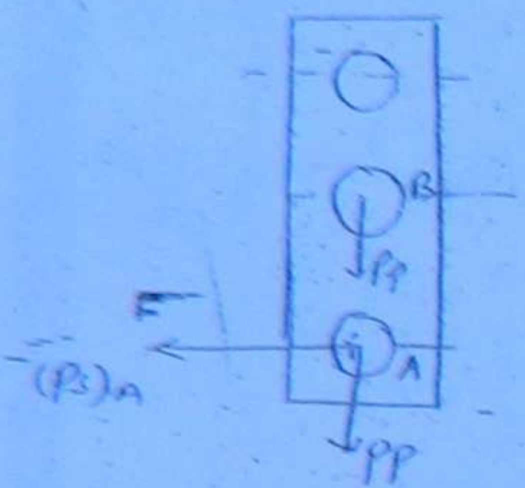
$\therefore d_c = ?$

for a bolted joint as shown in the fig. determine max. shear stress in bolts A and B



b) 242.6, 42.5 (c) 42.5, 42.5

a) 42.5, 242.6 (d) 242.6, 242.6




1/4

$P_P = \frac{P}{3}$

$R_B = P_P \quad [\because r_B = 0 \Rightarrow (P_s)_B = 0]$

$$\tau_B = \frac{4R_B}{\pi d_n^2} = 42.5 \text{ mpa}$$

$$R_A = \sqrt{P_P^2 + (P_s)_A^2}$$

$$\frac{(P_s)_A}{r_A^2}\left[2\, r_A^2 + 0^2\right] = P.e$$

$\therefore (P_s)_A = ?$

$$\tau_A = \frac{4R_A}{\pi (10)^2} = 242.6$$

# ⑦ WELDED JOINTS

→ permanent Joint

Advantages

① 100% Leak proof joint

② 98 to 100% efficiency is possible  115

$$\eta = \frac{\text{strength of riveted Joint}}{\text{strength of the solid plate}} \times 100$$

③ weight of welded Joint is less. (because of absence of no. of rivets and straps).

④ fatigue strength of welded Joint is more

⑤ Production time is less.

AIM: To determine the dimension of the weld which are obtained by.

DISADVANTAGES

① residual thermal stresses are developed

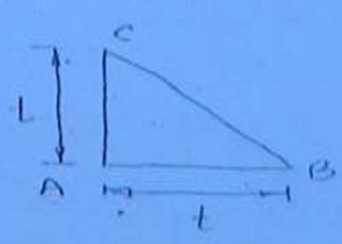
② grain structure is effected

③. skilled Labour is required

Types of welds used in fusion welding

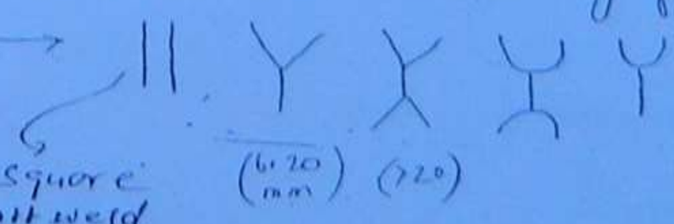
1) Tack welds → - - - - - -

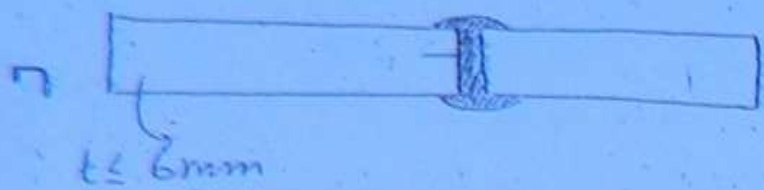
2) fillet welds →



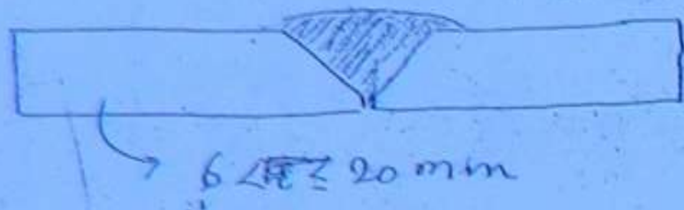

AB and AC are called leg of fillet weld

3) Butt welds →

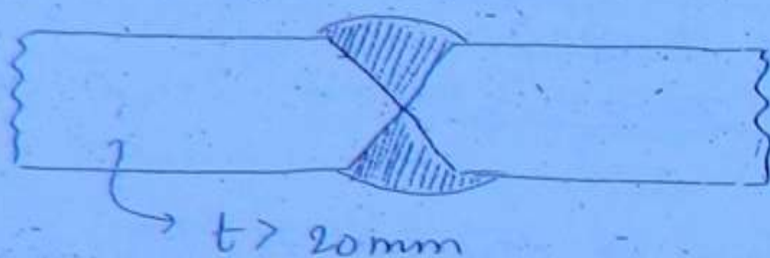





Square butt weld

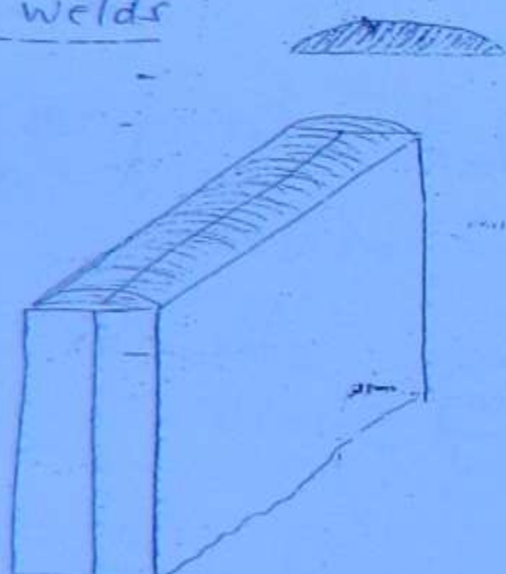



Single V butt weld

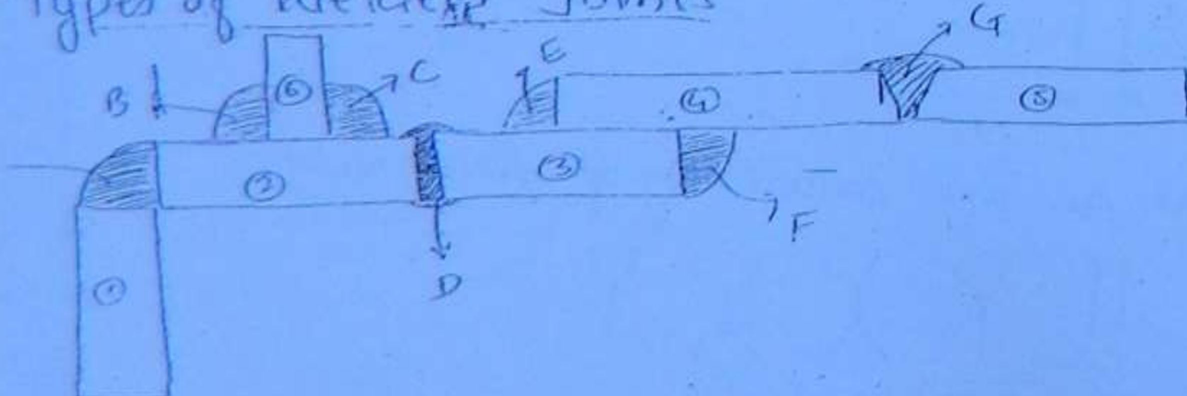



double V butt weld

Edge welds



Types of Welded Joints



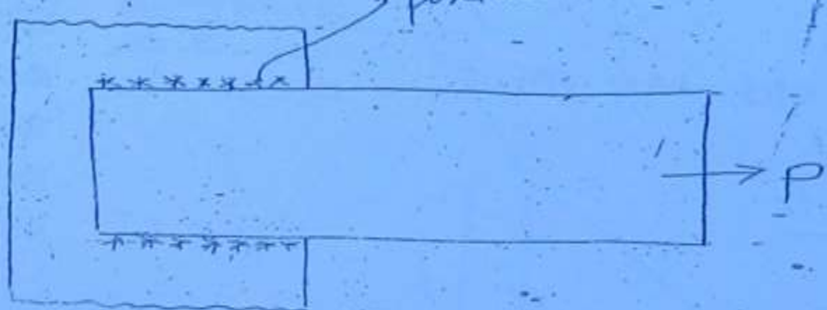
A, B, C, E, F ⇒ Fillet welds

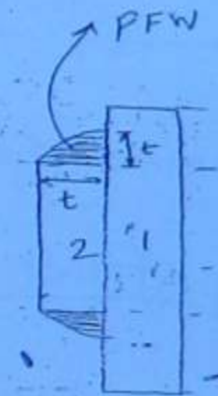
D and G ⇒ Butt welds

Types of fillet welds:

1. parallel fillet weld (PFW)
2. Transverse fillet weld (⊥ar) (TFW)
3. Compound fillet weld (CFW)

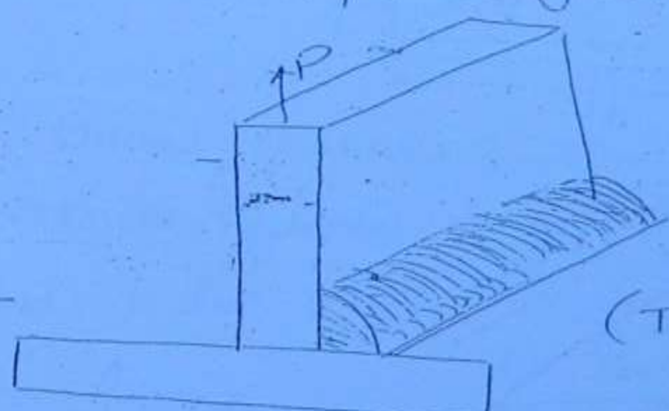
(1/7)



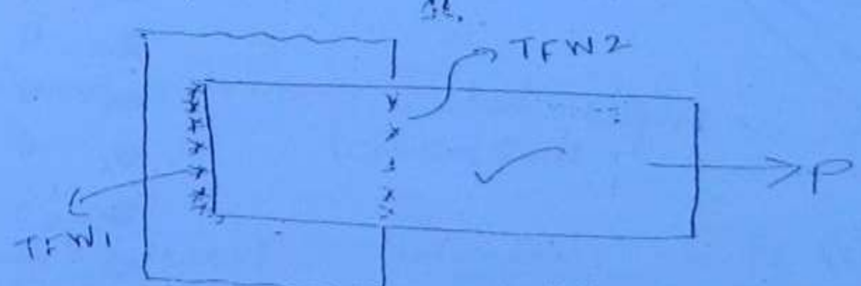




(F.V) (i) (S.V)

double parallel fillet weld




(TFW)



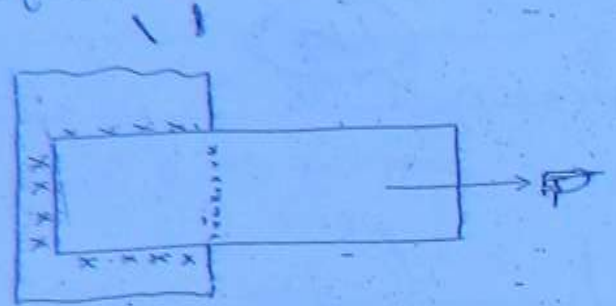

(ii)

DTFWLJ

(ii) is better than (i)

$P \leq P_{welds}$

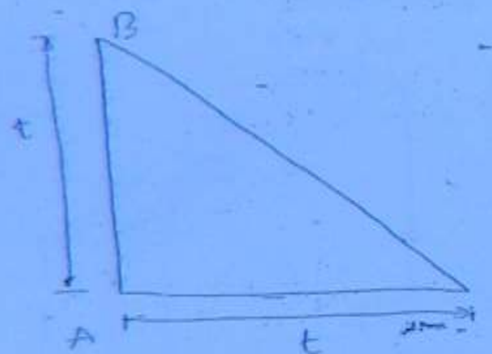
Note: Strength of TFW > strength of PFW
hence always welding is done ⊥ 'r to direction of Load.

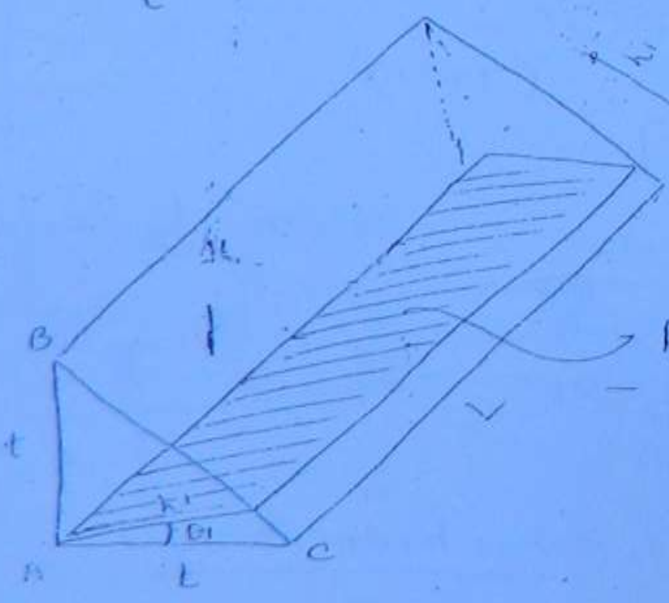


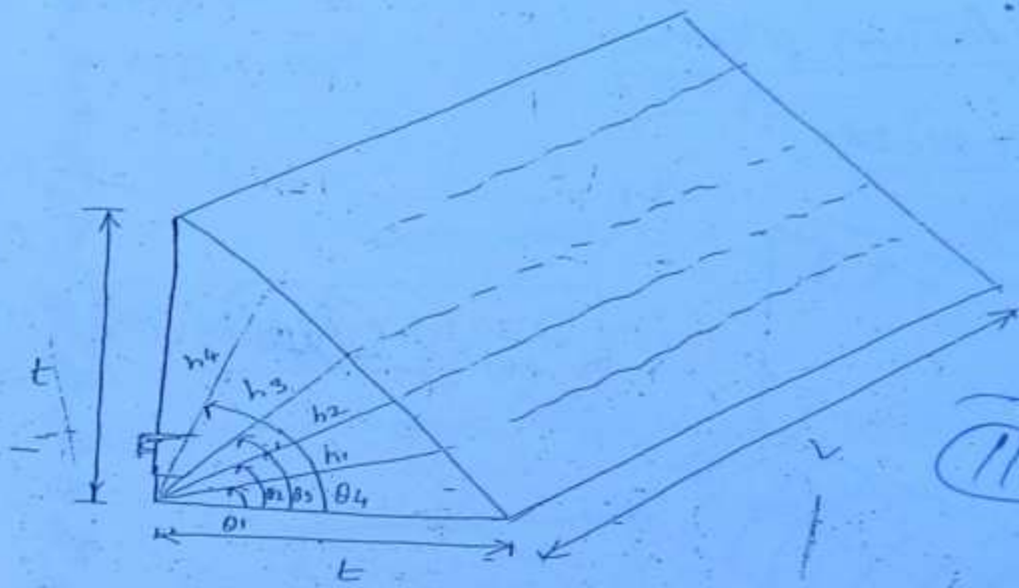
compound fillet weld. Lap Joint

(118)

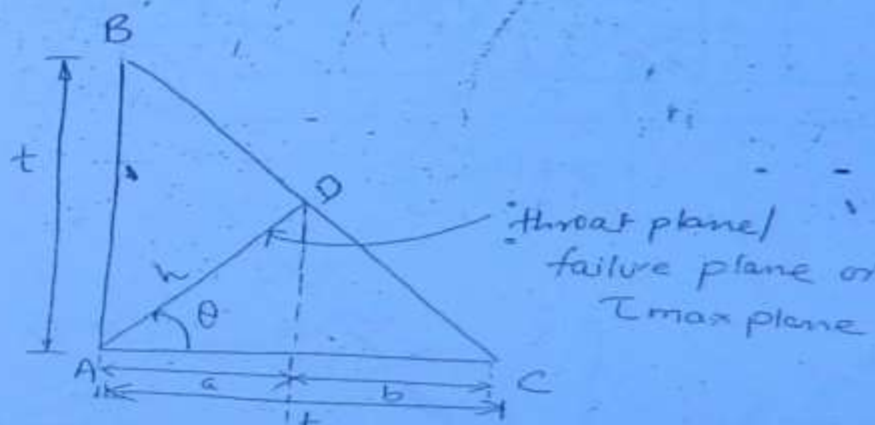
Strength of Fillet Welds.








h= throat thickness

thickness of weld along failure plane

$A_{weld} = h \, l_e$

$$h = \frac{t}{(\cos\theta + \sin\theta)} \qquad t = a + b$$

$$t = h\cos\theta + h\sin\theta$$

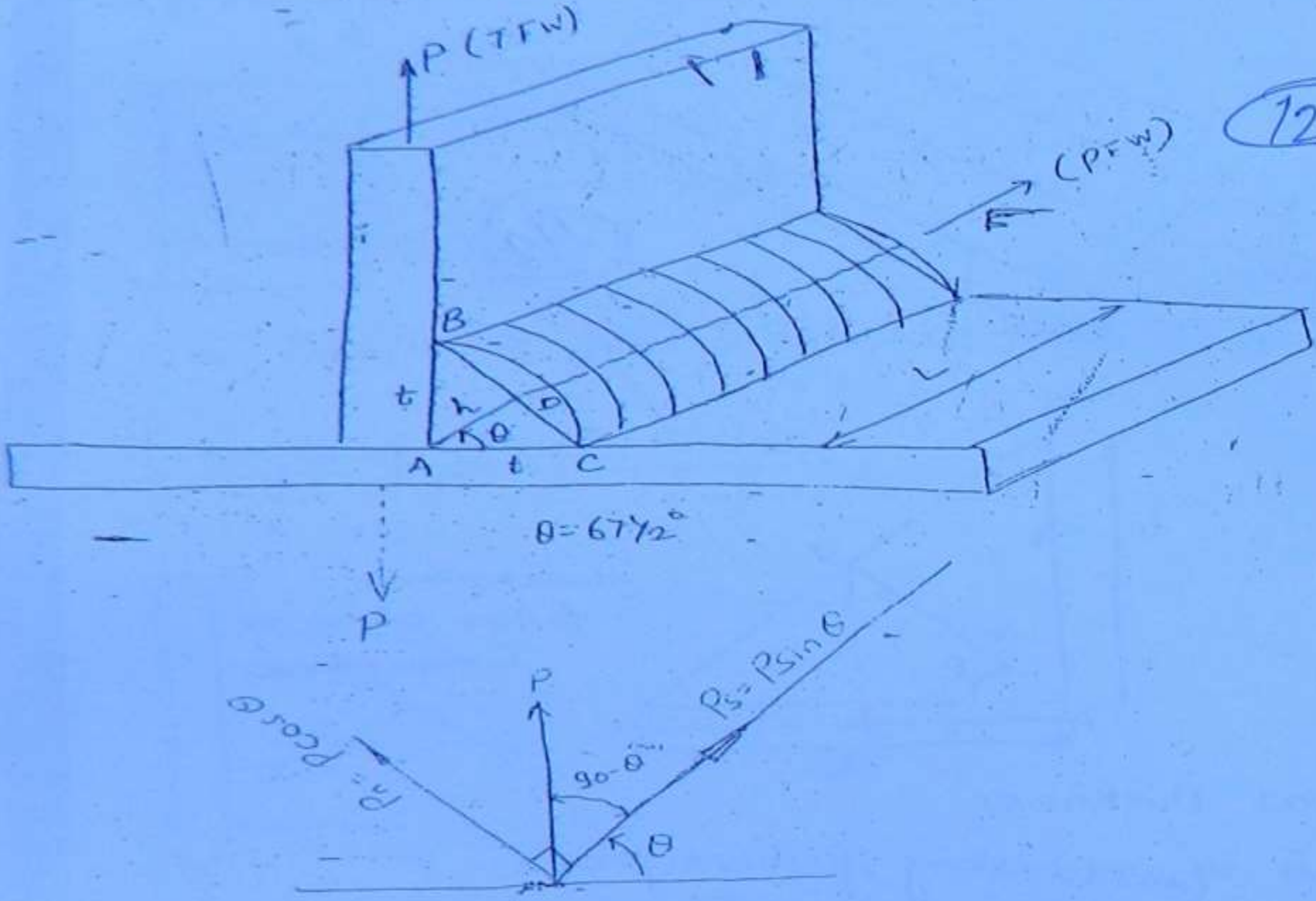
$$A_{weld} = h \cdot L_e = \left(\frac{t}{\cos\theta + \sin\theta}\right) L_e$$

$$(\tau_s)_{weld} = \frac{P_{shear force}}{A_{weld}} = \frac{P_s(\cos\theta + \sin\theta)}{t \cdot L_e}$$

(Q) Location of failure plane

(a) Transverse fillet weld

(120)




$$P_s = P\sin\theta$$

$$\tau_s = \frac{P\sin\theta\,(\cos\theta + \sin\theta)}{t \cdot L_e}$$

$$\frac{d\tau_s}{d\theta} = 0$$

$$\frac{d}{d\theta}\left[\frac{P\sin\theta\,(\cos\theta + \sin\theta)}{t \cdot L_e}\right] = 0$$

$$\frac{d}{d\theta}\left[\sin\theta\,(\cos\theta + \sin\theta)\right] = 0$$

$\tan 2\theta = -1$

$\therefore 2\theta = 135^\circ \quad \therefore \theta = 67\tfrac{1}{2}^\circ$

$A_{TFW} = h \cdot L_e = \left(\dfrac{t}{\cos\theta + \sin\theta}\right) l_e$

$\therefore \boxed{A_{TFW} = 0.765\, t \cdot L_e}$

(21)

$L_e = L \rightarrow$ single fillet welded joint

$L_e = 2L \rightarrow$ double fillet welded joint

<u>condition for safe design of Transverse fillet weld</u>

$(\tau_{max})_{ind} \leq \tau_{per}$

$\dfrac{P_s}{A_{TFW}} \leq \tau_{per}$

$\therefore \dfrac{P \sin\theta}{0.765\, t \cdot L_e} \leq \tau_{per}$ → constant

$\boxed{P \leq 0.828\, t \cdot L_e \cdot \tau_s} \Rightarrow$ strength of Transverse fillet weld

$P_{TFW} = 0.828\, t \cdot L_e \cdot \tau_s$

As per AWS (American welding society)

$\boxed{P_{TFW} = 0.83\overset{11}{2} \cdot t \cdot L_e \cdot \tau_s}$

→ parallel fillet weld (PFW)

$$\boxed{P_s = P, \quad P_n = 0}$$

$$\tau_s = \frac{P_s}{t \cdot L_e}(\sin\theta + \cos\theta)$$

$$\tau_s = \frac{P}{t \cdot L_e}(\sin\theta + \cos\theta)$$

$$\frac{d\tau_s}{d\theta} = \frac{P}{t \cdot L_e}\frac{d}{d\theta}(\sin\theta + \cos\theta) = 0$$

$$\tan\theta = 1, \quad \theta = 45$$

$$\boxed{\theta_{PFW} = 45}$$

$$h_{PFW} = \frac{t}{\cos\theta + \sin\theta} = \frac{t}{\sqrt{2}} = 0.707\,t$$

$$\boxed{A_{PFW} = 0.707\,t \cdot L_e}$$

for safe design of parallel fillet welds

$$(\tau_{max})_{ind} \leq \tau_{per}$$

$$\frac{P_s}{A_{weld}} \leq \tau_{per}$$

$$\frac{P}{0.707\,t \cdot L_e} \leq \tau_{per}$$

$$P \leq 0.707\,t \cdot L_e\,\tau_{per}$$

[illegible] PFW

$$P_{PFW} = 0.707\,t \cdot L_e \cdot \tau_s$$

$P$ [illegible] $t \cdot L_e\,\tau_s$

$$\frac{P_{TFW}}{P_{PFW}} = \frac{0.832 \cdot \cancel{t} \cdot \cancel{l_e} \cdot \cancel{\tau_s}}{0.707 \cancel{t} \cdot \cancel{l_e} \cdot \cancel{\tau_s}} = 1.18$$

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$\therefore \; P_{TFW} > P_{PFW}$

⇒ for a given dimension of weld and given weld material the strength of transverse fillet weld is 18% more than the strength of parallel fillet weld.

⇒ If unless otherwise mentioned it is better to assume the fillet weld as parallel fillet weld because it is the worst weld (i.e, the shear stress induced in PFW is more than the TFW)

or, ($\tau_{PFW} > \tau_{TFW}$)

| S.no. | parameter | PFW | TFW |
|---|---|---|---|
| 1. | direction of load | ∥el to direction of load | ⊥or to direction of Load |
| 2. | $P_s$ | $P$ | $P\sin\theta$ |
| 3. | $P_n$ | $0$ | $P\cos\theta$ |
| | $\theta$ | $45$ | $67\frac{1}{2}°$ |
| 5. | $h$ | $0.707\,t$ | $0.765\,t$ |
| 6. | $A$ | $0.707\,t\,l_e$ | $0.765\,t\,l_e$ |
| 7. | strength | $0.707\,t\,l_e\,\tau_s$ | $0.832\,t\,l_e\,\tau_s$ |

ES

1 The permissible stress in a fillet weld is 100 MPa. The fillet weld has equal leg lengths of 15 mm each. The allowable shear load on weld per cm length of weld is

(a) 22.5 kN (b) 15 kN (c) 7.5 kN (d) 10.6 kN

default PFW

$P_{FW} = 0.707\ t \cdot L_e \cdot \tau_s$

$= 0.707 \times 15 \times 10 \times 100$

$= 10.6 \times 10^3 N$

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2 A double fillet welded joint with parallel fillet weld of length 'l' and leg 'b' is subjected to tensile force P. Assuming uniform stress distribution, the shear stress in the weld is given by

(a) $\frac{\sqrt{2}P}{b.l}$ (b) $\frac{2P}{bl}$ (c) $\frac{P}{2bl}$ (d) $\frac{P}{\sqrt{2}\,bl}$

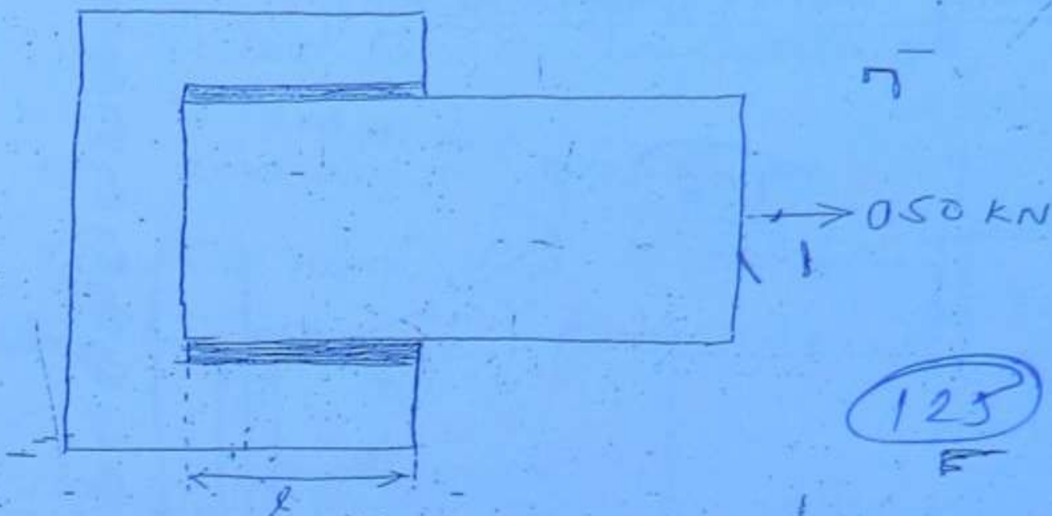
Sm $P = 0.707\ b \cdot 2l\ \tau_s$

$= \frac{1}{\sqrt{2}}\ b \times 2l \times \tau_s$

$\therefore \tau_s = \frac{P}{\sqrt{2}\,b \cdot l}$

The two plates are joined by means of fillet welds as shown in figure. The size of the fillet weld is 10 mm and the allowable shear stress is 75 MPa. The length of the weld is

(a) 47 mm (b) 55 mm (c) 45 mm (d) 100 mm

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Soln $P \le P_{weld}$

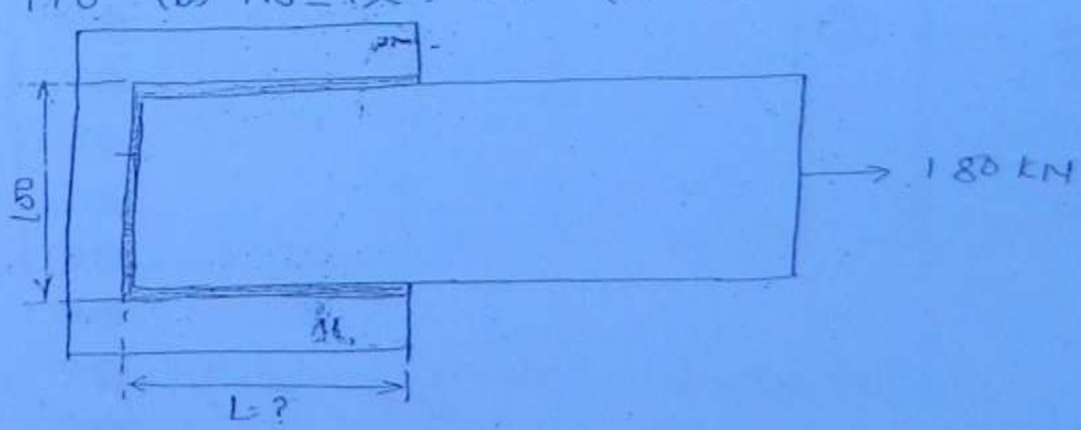
$$50\times10^3 \le 0.707\times10\times 2\ell\times 75$$

$$\therefore \ell \ge 47.14$$

Imp

Q.4 Two plates are joined together by means of single transverse and double parallel fillet welds as shown in the figure the size of fillet weld is 5mm and allowable shear load per mm of load weld is 400 N find the length of each parallel fillet weld?

(a) 170 (b) 175 (c) 185 (d) 225



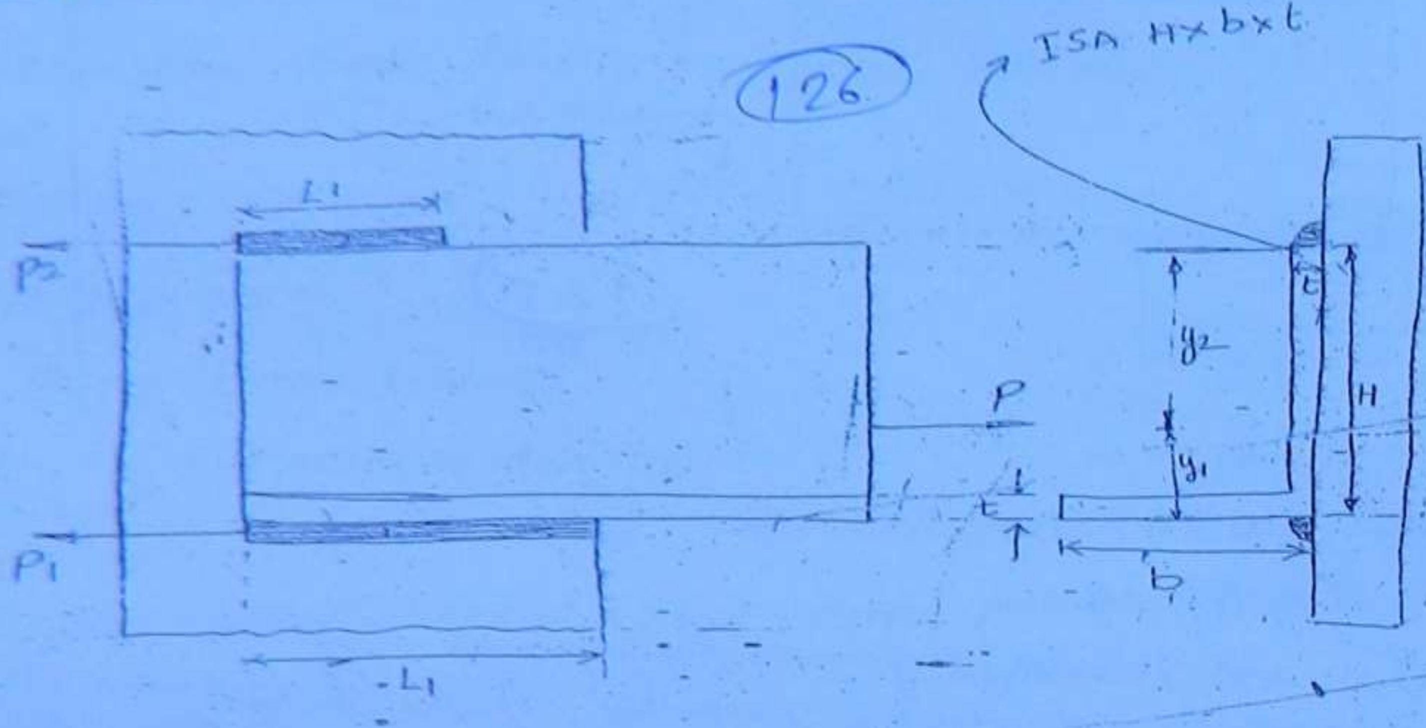

Soln Total length of weld = $2L+100 = \dfrac{\text{Total Load}}{\text{Allowable shear load/mm}}$

$$2L+100 = \frac{180\times10^3}{400}$$

$$\therefore L = 175$$

## Fillet welding of axially loaded unsymmetrical sections






$$P = P_1 + P_2$$

$$P = 0.707 \times t \times L_1 \times \tau_s + 0.707 \times t \times L_2 \times \tau_s$$

$$P = 0.707\, t\, \tau_s \,[L_1 + L_2]$$

$$L_1 + L_2 = ? \longrightarrow \text{(I)}$$

$$\Sigma M = 0$$

$$\therefore P_1 y_1 - P_2 y_2 = 0$$

$$P_1 y_1 = P_2 y_2$$

$$\cancel{0.707} \times \cancel{t} \times L_1 \times \cancel{\tau_s} \times y_1 = \cancel{0.707} \times \cancel{t} \times L_2 \times \cancel{\tau_s} \times y_2$$

$$L_1 y_1 = L_2 y_2$$

$$\therefore \frac{L_1}{L_2} = \frac{y_2}{y_1} \longrightarrow \text{(II)}$$

Solving I and II we can get
$L_1$ and $L_2$. can be calculated.

Q: ISA 200×100×10, $P = 150$ kN
$\tau_{weld} = 75$ mpa find $L_1$ and $L_2$
also, $y_1 = 71.8$; $y_2 = (200 - 71.8)$.

$L_1 = 194.98$ mm
$L_2 = 108.81$ mm

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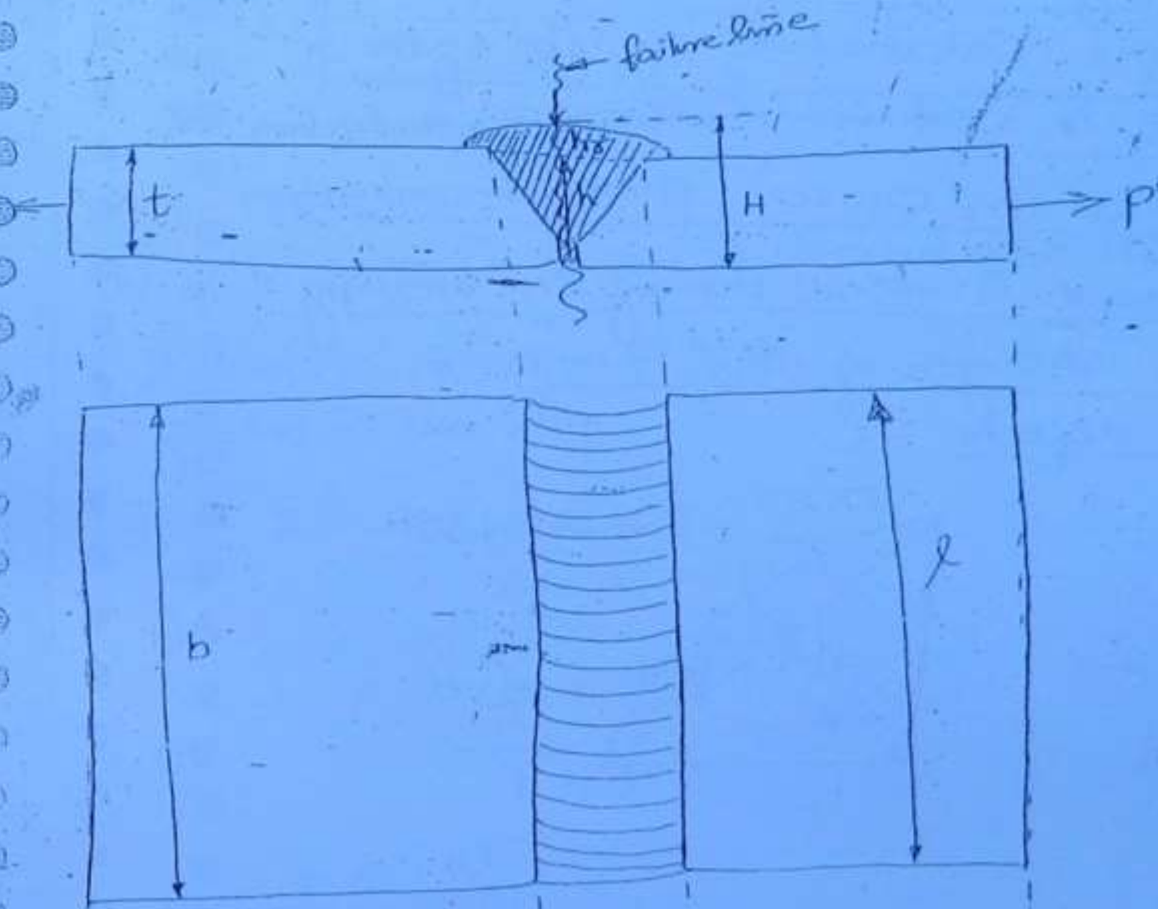
DESIGN OF BUTT WELDS




for safe design of welds

$(\sigma_{max})_{ind} \leq (\sigma_{per})_{weld\ material}$

$$\frac{P}{A_{weld}} \leq (\sigma_t)_{per}$$

$\therefore \dfrac{P}{h \cdot l} \leq (\sigma_t)_{per}$

$\therefore P \leq (\sigma_t)_{per} \times h \times l$

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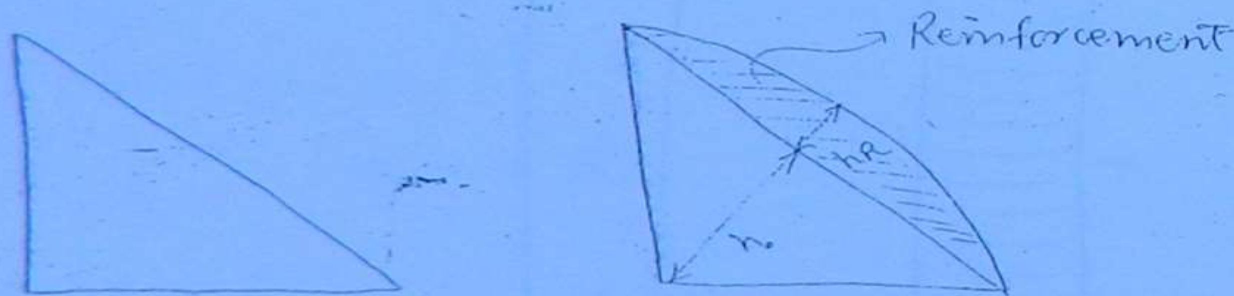
Now in fig. $h = t$, $l = b$

$h_R$ = height of Reinforcement of Welds

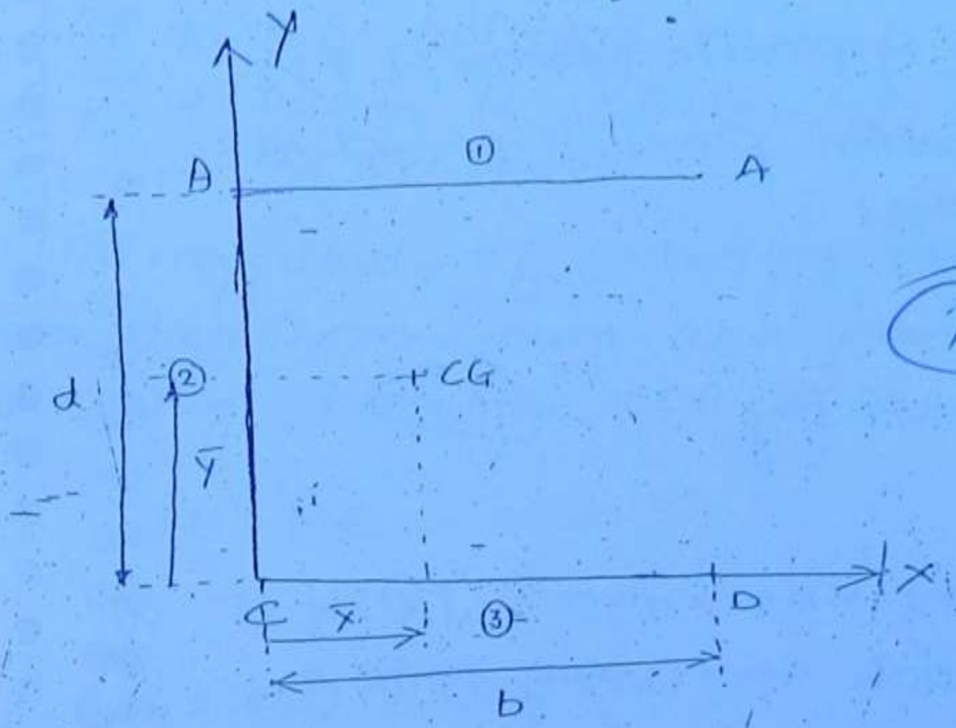
Reinforcement height is not taken into the
nsideration in the Calculation of strength
f weld because it ~~causes stress concentration~~
s grinded off as it causes stress concentration

Reinforcement is done during the welding
o compensate strength of the welds in
esence of weld defects





$\boxed{h = \text{min of } [t_1 \text{ and } t_2]}$




Let C.G of weld system is located at a distance of $\bar{y}$ and $\bar{x}$ from y and x axis respectively as shown in the fig

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$$

$$= \frac{b \times \frac{b}{2} + d(0) + b/2}{b + d + b}$$

$$\bar{x} = \frac{b^2}{2b+d}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{b \times d + d \times \frac{d}{2} + b(0)}{b + d + b}$$

$\bar{y} = d$

Introduce two equal and opposite forces $P_1$ & $P_2$ through C.G in a direction parallel to applied load in such a way that

$$P_1 = P_2 = P$$

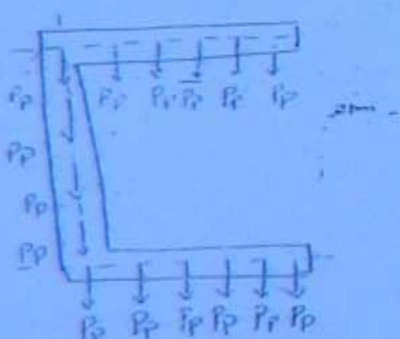
determination of ecentricity (1.30)

- Effect of $P_1$

is to cause a primary shear force ($P_P$) of same magnitude at each and every point on the weld system.

$$P_P = \frac{P_1}{\sum L_{weld}} = \frac{P}{2b+d} \text{ N/mm of weld}$$

$$\tau_P = \frac{P_P}{A_{weld}} = \frac{P_P}{0.707 \times t \times L \text{ mm}} \text{ MPa}$$



or, $$\tau_P = \frac{P_1}{A_{weld}} = \frac{P}{0.707 \cdot t \times (2b+d)} \text{ MPa}$$

5) effect of P and $P_2$

It causes a twisting moment

$$TM = P \cdot e$$

due to this twisting Moment each point on the weld system is subjected to a secondary torsional shear stress.

The magnitude of secondary ~~stre~~ torsional Shear stress is maximum at a point which is far away from the C.G of weld system.

$TM = p \cdot e$

As $\tau_s \propto r$

$(r_A = r_D) > (r_B = r_C)$

$[(\tau_s)_A = (\tau_s)_D] > [(\tau_s)_B = (\tau_s)_C]$

$(\tau_s)_{max} = (\tau_s)_A$ or $(\tau_s)_D$

$$\tau_s = \frac{T}{Z_P} = \frac{T}{J/r} = \frac{T \cdot r}{J_{weld}}$$

$$(\tau_s)_{max} = \frac{T(r_A \text{ or } r_D)}{J_{weld}} = ?$$

Where $J_{weld}$ polar moment of inertia of the entire weld system about the C.G of weld system

$J_{weld} = J_{G1} + J_{G2} + J_{G3} + \cdots$

6) $(\theta_A = \theta_D) < (\theta_C = \theta_B)$

$(\tau_R)_{max}$ where $\theta$ is minimum

and $\tau_s$ is max

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$$(\tau_R)_{max} = (\tau_R)_A \text{ or } (\tau_R)_D$$

using parallelogram law.

$$\tau_R = \sqrt{\tau_P^2 + \tau_S^2 + 2\tau_P \tau_S \cdot \cos\theta}$$

$$(\tau_R)_{max} = \sqrt{(\tau_P)^2 + (\tau_S)^2_{max} + 2\tau_P(\tau_S)_{max} \cos\theta_A}$$

$$= \frac{?(x)}{t} \text{ MPa}$$

(132)

Design of fillet welds

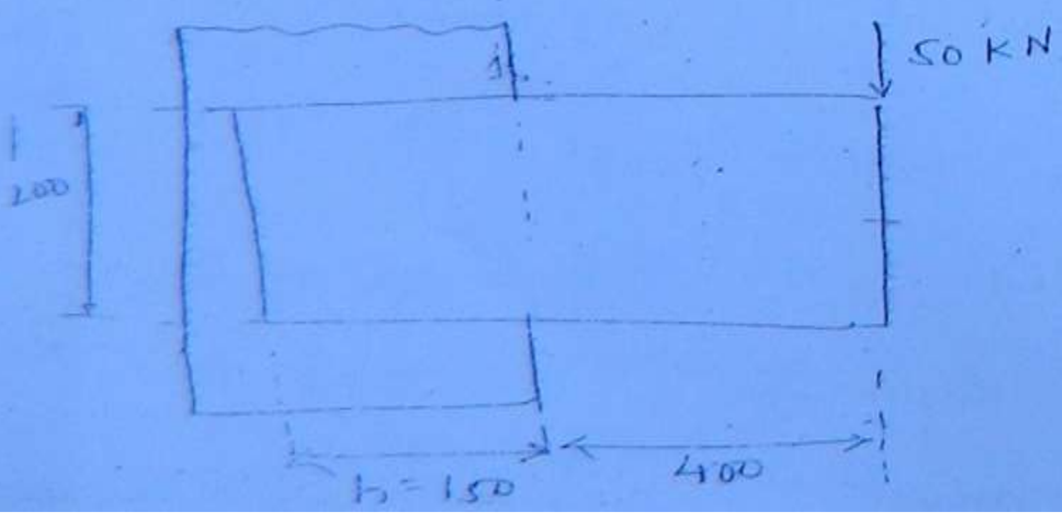
Safe design

$$(\tau_R)_{max} \le \tau_{per}$$

$$\frac{x}{t} \le (\tau_{per})_{weld}$$

$$t \ge \_\_ \text{ mm}$$

fig shows an eccentrically loaded welded joint determine the fillet size if $(\sigma_t)_{per}$ is 80 MPa



$$J_w = \left[\frac{(2b+d)^3}{12} - \frac{b^2(b+d)^2}{2b+d}\right]\underset{=}{h} \rightarrow 0.707\,t$$

$$= \text{-------} \; mm^4$$

$$\bar{x} = \frac{b^2}{2b+d} = 45\ mm$$

$\bar{y} = 100\ mm$

$e = 505\ mm$

$$\tau_p = \frac{P}{0.707 \times t \times le} = \frac{50 \times 10^3}{0.707 \times t \times 500} = \frac{141.4}{t}\ mpa$$

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$r_{max} = r_A$ or $r_D = 145\ mm$

$J_w = 3468.3 \times 10^3\, t\ mm^4$

$$(\tau_s)_{max} = (\tau_s)_A \text{ or } (\tau_s)_D = \frac{T \times r_{max}}{J_{weld}}$$

$$= \frac{105.56}{t}\ mpa$$

$\theta_{min} = \cancel{\theta_A}$ or $\theta_D$

$-\cos\theta_{min} = 0.724$

$$(\tau_R)_{max} = \frac{1162.1}{t}\ mpa \leq 80$$

$t > 14.5\ mm$

Case II (Moment is acting in a plane ⊥ or to the plane of welds)
($\tau_s$ and $\tau_b$)

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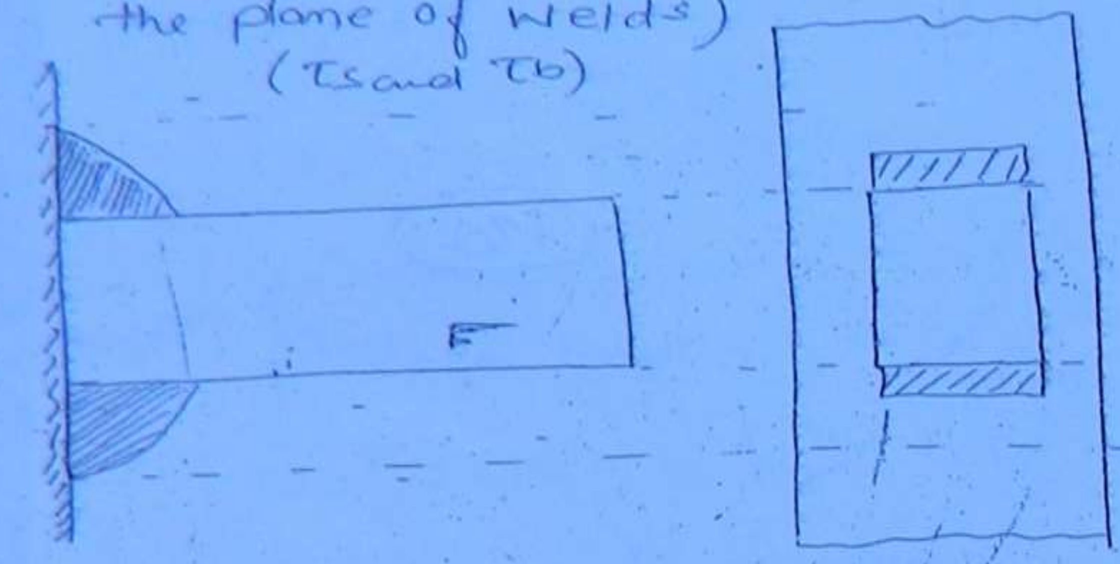


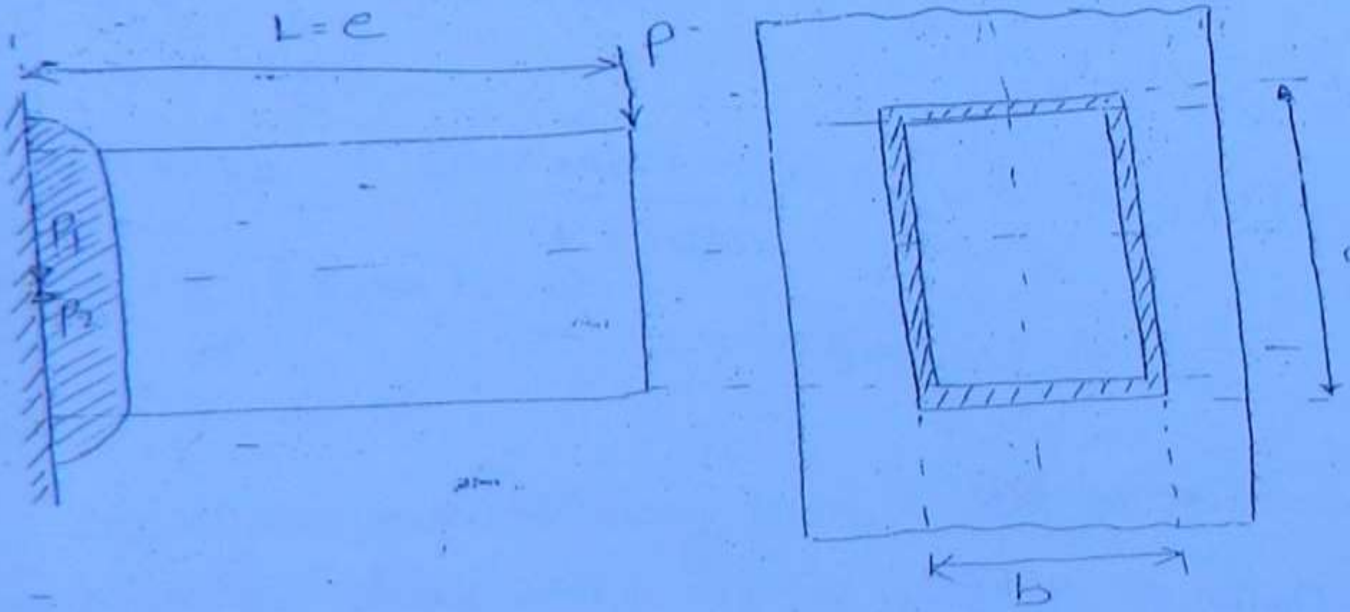


Effect of $P_1$
is to cause a shear stress of same magnitude at each and every point on the weld

$$\tau_s = \frac{P_1}{A_w} = \frac{P}{0.707\,t\,l_c} = \frac{P}{0.707 \times t \times (2b+2d)} = \frac{X}{t}\ \text{MPa}$$

ii) Effect of P and $P_2$

$$M = P \cdot e = P \times L$$

due to this bending moment as the bar is subjected to bending the welds also subjected to bending stresses.

$$\sigma_b = \frac{M}{Z_{weld}} = \frac{Y}{t} \text{ MPa} \quad \rightarrow (2)$$

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where $Z_w = \frac{I_w}{Y_{max}} = \frac{\quad}{t}$ MPa/mm$^3$

iii) design of fillet weld

Here fillet welds are designed by using maximum shear stress theory or maximum Distortion energy theory as fillet welds are subjected to combined stresses.

MSST $\Rightarrow$ $\tau_s = \frac{S_{ys}}{N} = \frac{1}{2}\sqrt{\sigma_b^2 + 4\tau_s^2}$

$$= \frac{1}{2}\sqrt{\left(\frac{Y}{t}\right)^2 + 4\left(\frac{X}{t}\right)^2}$$

$$\therefore t \geq \text{ — mm}$$

MDET $\Rightarrow$ $\sigma_t = \frac{S_{yt}}{N} = \sqrt{\sigma_b^2 + 3\tau_s^2}$

$$= \sqrt{\left(\frac{Y}{t}\right)^2 + 3\left(\frac{X}{t}\right)^2}$$

$$t \geq \text{ mm}$$

Case-3


$$Z_w = \frac{\pi d^2}{4} \times h$$

$$J_w = \frac{\pi d^3}{4} \times h$$

Fillet welds under pure Torsion

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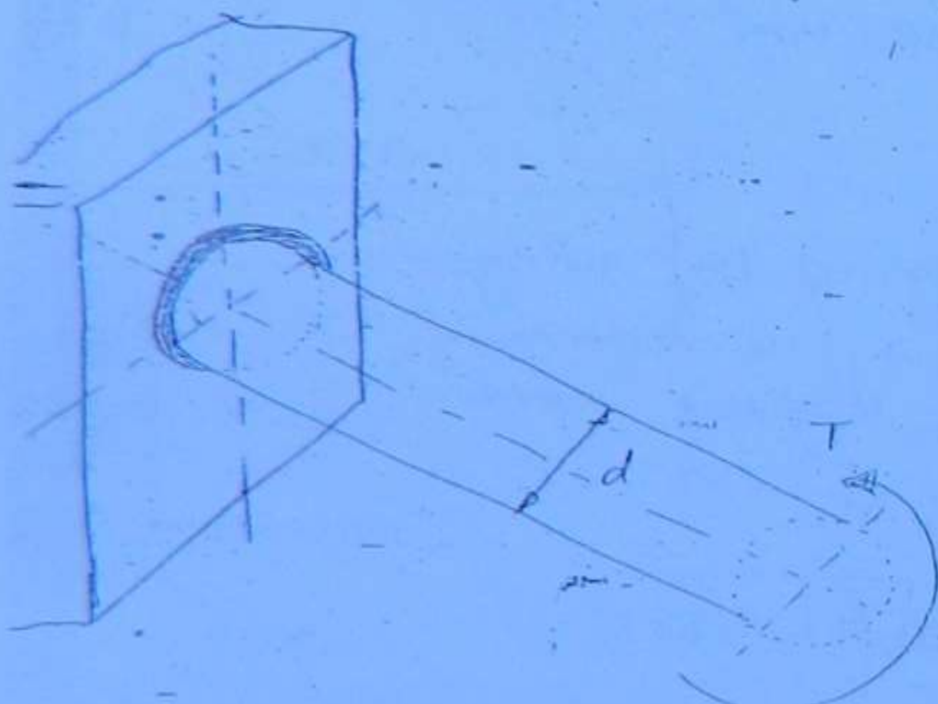
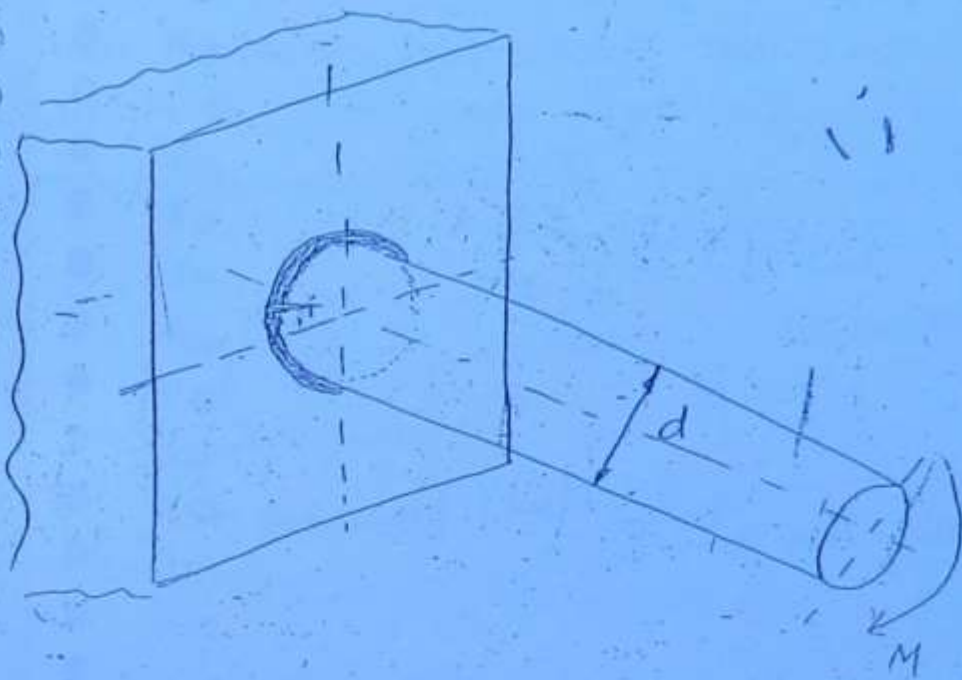



$$\tau_s = \frac{T}{Z_p} = \frac{T \cdot r}{J_w}$$

$$= \frac{T \cdot d/2}{\frac{\pi d^3}{4} \times \frac{t}{\sqrt{2}}} = \frac{4\sqrt{2}\, T \cdot d}{2\pi d^3 t}$$

$$\boxed{\tau_s = \frac{2.83 T}{\pi d^2 t}}$$

Case-4 fillet weld under pure bending

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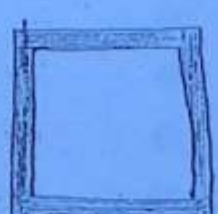
$$\sigma_b = \frac{M}{Z}$$

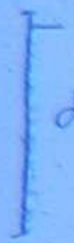
$$= \frac{M}{\frac{\pi d^2}{4} \times \frac{t}{\sqrt{2}}} = \frac{4\sqrt{2}\, M}{\pi d^2 t}$$

$$\boxed{\sigma_b = \frac{5.66\, M}{\pi d^2 t}}$$

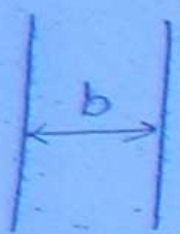


$$Z_w = \left(bd + \frac{d^2}{3}\right) h$$

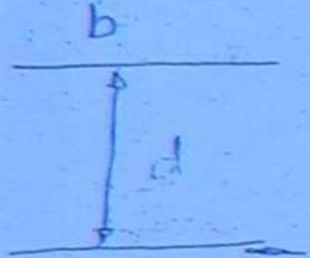
$$J_w = \frac{(b+d)^3}{6} \cdot h$$




$$Z_w = \frac{d^2}{6}h, \quad J_w = \frac{d^3}{12}h$$




$$Z_w = \frac{d^2}{3}h, \quad J_w = d\,\frac{(3b+d^2)}{6}\,h$$




$$Z_w = bdh, \quad J_w = \left(\frac{b^3+3bd^2}{6}\right)h$$

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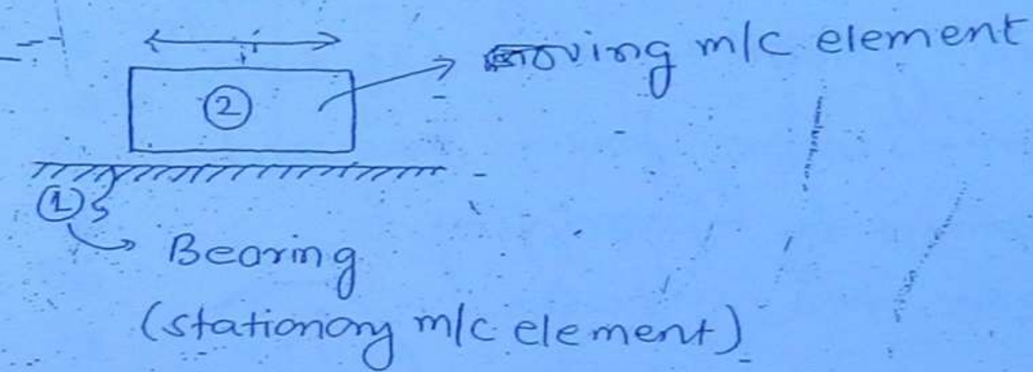
# ⑧ BEARINGS

* Whenever relative motion takes place between Two machine elements the machine element which is stationary and supporting the moving machine element is called as a bearing

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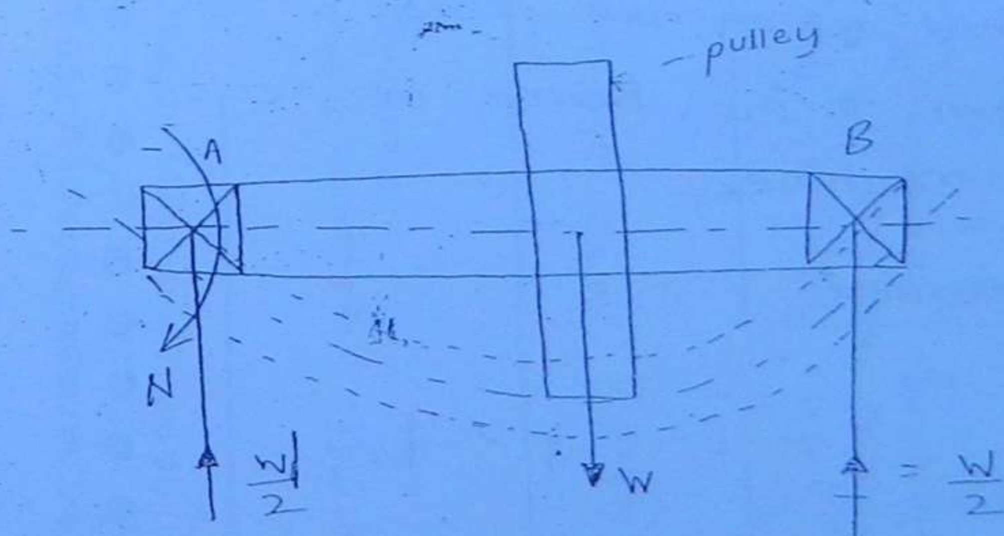
⇒ Bearing is defined as a machine element whose function is to support a rotating machine element (i.e. a shaft) and to guide or confines its motion, while preventing its motion in the direction of applied Load



because of relative motion between the shaft and bearing surfaces, always some amount of power loss takes place in overcoming the frictional resistance and wear of the surfaces takes place due to metal to metal contact hence a bearing is said to be a good bearing which performs its given function (ie, supporting the shaft) with minimum power loss and wear, this is obtained by providing lubrication between two surfaces.

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Functions of bearing—

To support the shaft and axle and holds in it correct position

It ensures free rotation of the shaft and axle with minimum friction

Takes up loads that act on the shaft and transmits them to the frame or foundations of the machine.

Classification of bearings

$P_{loss} = \mu WV$

$\mu_{rolling} << \mu_{sliding}$

$(P_{loss})_{mRCB} <<< (P_{loss})_{SCB}$

and hence called as antifriction bearing.

- Sliding contact bearing (SCB)
  - nature of sliding action
    - Sleeve bearings
      - w.r.t direction of load
        - Thrust bearing (eg- foot step bearing)
        - Radial bearings
      - w.r.t Lubrication
        - Thick film Lubrication
          - Hydrodynamic Lubrication
          - Hydrostatic Lubrication (externally pressurised Lubrication)
        - Thin film or boundary Lubrication
        - Zero film Lubrication or (Self Lubricating bearing)
      - w.r.t to configuration
        - Solid bush bearing
        - Split bush bearing
    - Slipper or guide bearings
- Rolling contact bearing (RCB) [Anti-friction bearing]
  - Shape of rolling elements
    - Ball bearing
    - Roller bearing (RB)
      - Cylindrical RB
      - Tapered RB
      - Spherical RB
      - Needle RB


Sliding contact bearing (liquid Lubricants)

urnal = a portion of shaft which is inside the bearing is called Journal

$\{ L_{Journal} = L_{bearing} \}$ L = Length

$(\text{Diameter})_{Journal} = (\text{Diameter})_{bearing}$

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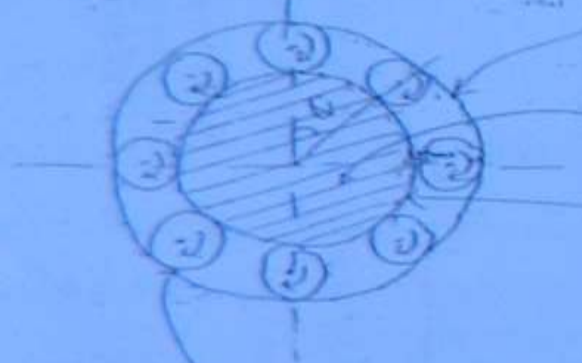
olling contact bearing

ere semi solid Lubricants are used.

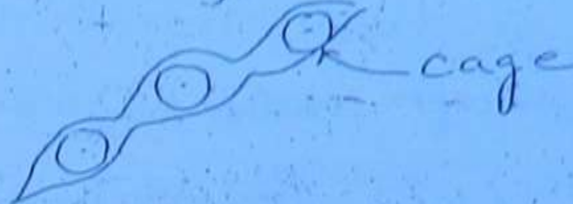



nction of cage or seperator -

To prevent clustering of rolling elements

To maintain relative angle between the adjacent rolling elements or evenly spaced

3. To avoid contact or to seperate the adjacent rolling elements.




⇒ In case of hollow shaft outer race will be moving and inner race will be stationary.

Roller bearings

plane → cylindrical roller 1.43

Tapered → Tapered roller

spherical → spherical roller

$\boxed{L/d < 1}$

Needle → $\boxed{L/d > 1}$ → Needle roller

1. It is used when radial space is constrained
2. There is no cage in Needle roller bearing

Sleeve bearing: sliding action along an arc of circle.

slipper or guide bearing: sliding action along a straight line eg (lathe)

Radial bearings: They are used to supports the radial loads (ie, loads ⊥or to the shaft axis.

Thrust bearings: Support a shaft where load is acting along the shaft axis), eg footstep bearing

Zero film Lubrication - materials are having property of self lubricating. eg CI and graphite

⇒ generally bushes are provided between shaft and bearing (they are splitpeace-type).

Split bush bearing: eg plummer block

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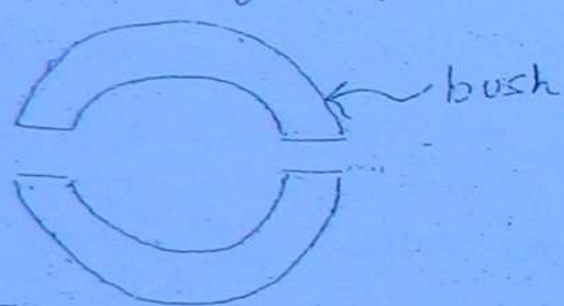



⇒ plummer block is used to support a lengthy shaft which requires support at intermediate locations

# Thrust bearings (TB)

They are used to support a shaft which is subjected to thrust loads ie, Loads acting on along the shaft axis.



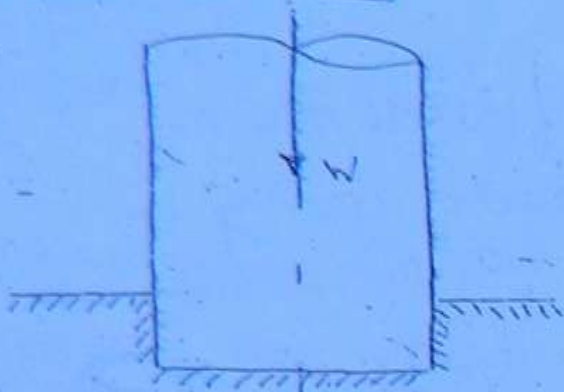
- Thrust bearings
  - pivot bearing (PB) (vertical shaft)
    - flat pivot bearings (foot step bearing)
    - conical pivot bearings
  - Collar bearing [CB] (horizontal shaft)
    - flat collar bearing
      - Single collar bearing
      - Multi collar bearing
    - Conical collar bearing

## Collar bearing

$R_i$ = shaft radius

To get equation of pivot bearing substitute $R_i = 0$ and $R_o = R$ in the equation of collar bearings.

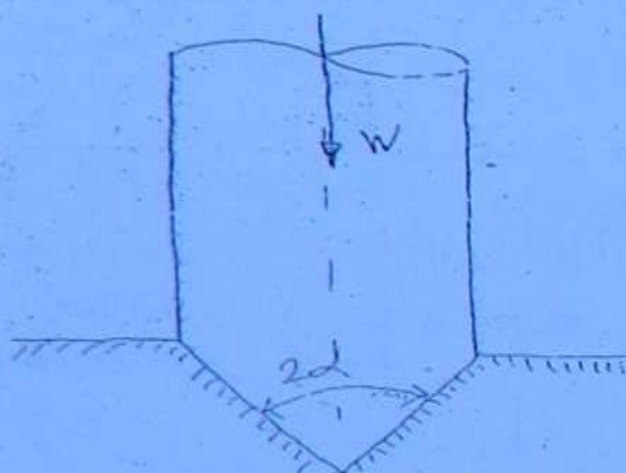
pivot bearing




foot step bearing

flat pivot bearing

used to support vertical shaft subjected to thrust load.

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conical pivot bearing

$2\alpha$ = cone angle

$\alpha$ = semi cone angle

When $\alpha = 90^\circ$ conical pivot bearing becomes as flat pivot bearing

# Collar bearing

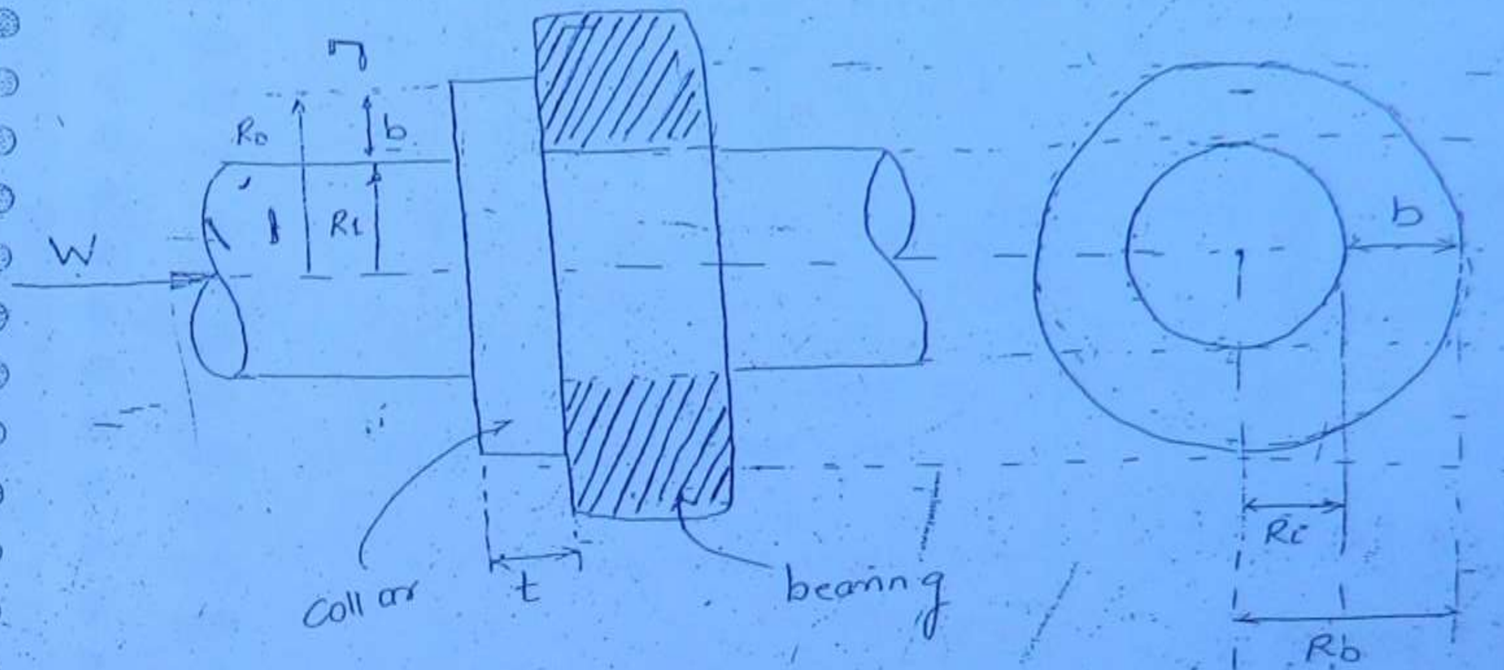



fig: Flat collar bearing.
(or single collar bearing)

(147)

$$p = \frac{W}{\pi (R_o^2 - R_i^2)}$$

$$W = p\pi (R_o^2 - R_i^2)$$

$W \Rightarrow R_o = \cancel{x}\,mm$

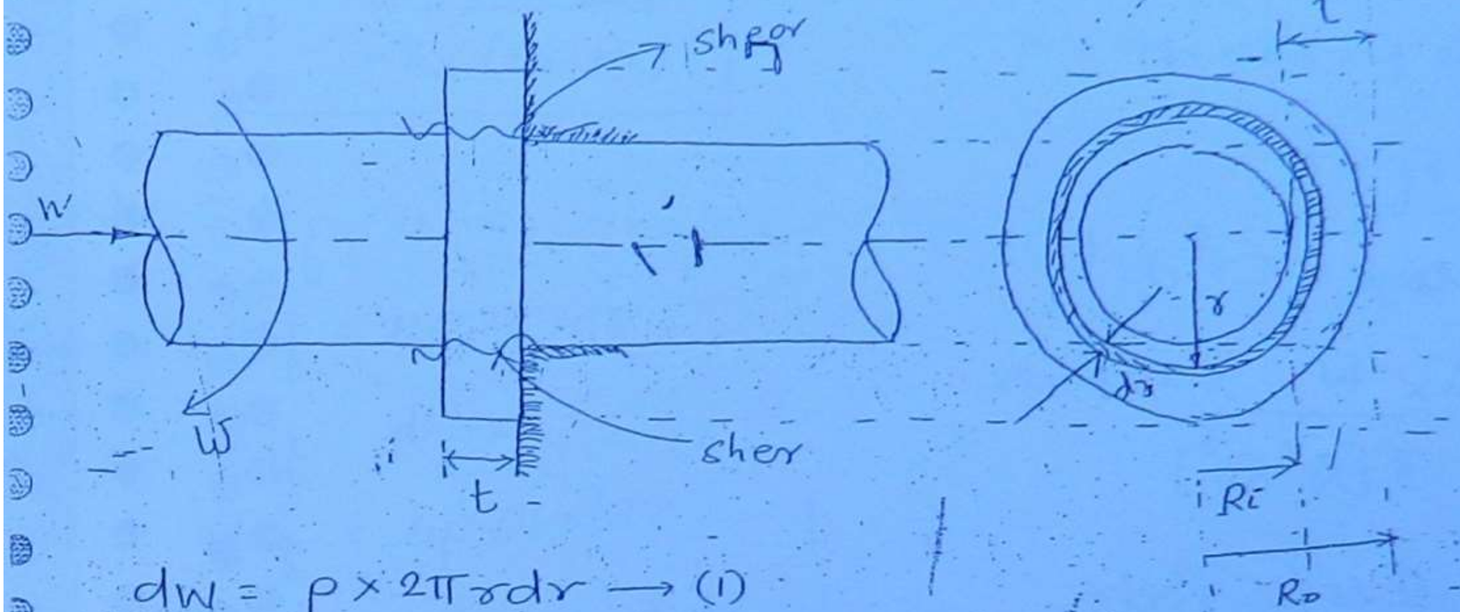
$2W \Rightarrow R_o = 2x\ mm$

Design equation used in Thrust bearings
[P, W and $T_f$]

Uniform pressure theory [Pressure = constant]
(UPT)

and 2nd uniform wear theory
(UWT)

## SINGLE COLLAR BEARING




$$dW = p \times 2\pi r\,dr \longrightarrow (1)$$

$$W = \int_{Ri}^{Ro} dW = \int_{Ri}^{Ro} p \times 2\pi r\,dr \longrightarrow (2)$$

T.49

as per uniform pressure theory $(p = c)$

$$W = p \times 2\pi \int_{Ri}^{Ro} r\,dr$$

$$W = p \times 2\pi \times \frac{Ro^2 - Ri^2}{2}$$

$$\boxed{W = \pi \times p\,(Ro^2 - Ri^2)} \longrightarrow (3)$$

as per UPT

$$\boxed{P_{UPT} = \frac{W}{\pi\,(Ro^2 - Ri^2)}} \longrightarrow (4)$$

$$dF_f = \mu \cdot dW$$

$$= \mu \cdot p \times 2\pi r \, dr$$

$$dT_f = dF_f \times r$$

$$= \mu \cdot p \, 2\pi r^2 dr$$

$$dT_f = \mu \times \frac{W}{\pi [R_o^2 - R_i^2]} \times 2\pi r^2 dr$$

$$dT_f = \frac{2 \mu W}{R_o^2 - R_i^2} r^2 dr$$

$$T_f = \int_{R_i}^{R_o} \frac{2 \mu W \cdot r^2 dr}{R_i^2 - R_i^2}$$

$$T_f = \frac{2 \mu W}{R_o^2 - R_i^2} \times \frac{R_o^3 - R_i^3}{3}$$

$$\boxed{T_f = \frac{2}{3} \mu W \left[ \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} \right]}$$

for uniform wear theory

$$p r = \text{constant}$$

$$W = 2\pi p r \int_{R_o}^{R_o} dr$$

$$W = 2\pi p \cdot r \, (R_o - R_i)$$

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$$\therefore \boxed{P = \frac{W}{2\pi r(R_o - R_i)}}$$

Now, $dF_f = \mu dw$

$$dF_f = \mu p \, 2\pi r \, dr$$

$$dT_f = dF_f \times r$$

$$dT_f = \mu p \, 2\pi r^2 \, dr$$

$$dT_f = \frac{\mu W \times \cancel{2\pi r^2} \, dr}{\cancel{2\pi r}(R_o - R_i)}$$

$$\therefore dT_f = \frac{\mu W}{R_o - R_i} r \, dr$$

$$T_f = \frac{\mu W}{R_o - R_i} \int_{R_i}^{R_o} r \, dr$$

$$T_f = \frac{\mu W}{R_o - R_i} \times \frac{R_o^2 - R_i^2}{2}$$

$$\boxed{T_f = \mu W \left[\frac{R_o + R_i}{2}\right]}$$

Thickness of collar is determined by considering shear failure of collar at inner radius

for safe design

$$(\tau_{max})_{ind} \leq \tau_{per}$$

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$$\frac{F_s}{A_s} \leq \tau_{per}$$

$$\frac{W}{\pi D_i t} \leq \tau_{per}$$

$\therefore t \geq$ — mm

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**rictional torque equation for flat Pivot bearing r footstep bearing :—**

he equations for the flat pivot bearing is putting $R_i = 0$ and $R_o = R$ in the equations of Single collar bearing.

$$(T_f)_{UPT} = \frac{2}{3} \mu W \left( \frac{R^3 - 0}{R^2 - 0} \right)$$

$$\boxed{(T_f)_{UPT} = \frac{2}{3} \mu W R} \quad \text{— (i)}$$

$$(T_f)_{UWT} = \frac{1}{2} \mu W (R + 0)$$

$$\boxed{(T_f)_{UWT} = \frac{1}{2} \mu W R} \quad \text{— (ii)}$$

$$\frac{(T_f)_{UPT}}{(T_f)_{UWT}} = \frac{4}{3} = 1.33$$

$\therefore$ UPT gives more power loss and hence alway design for UPT default

⇒ from above two equation we can conclude that frictional torque or power loss as per uniform pressure theory is more than the power loss or frictional loss as per UWT, hence for the safe design of bearing, If unless otherwise mention it is better to use uniform pressure theory because always power loss takes place in bearings due to frictional forces.

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⇒ For the safe design of clutches (old or worn out clutches) It is better to use uniform wear theory because clutches are used to transmit power by utilising frictional forces and when the clutches are come into service pressure is not uniformly distributed.

For the safe design of New clutches it is better to use uniform pressure theory because pressure is uniformly distributed when the clutch surfaces are new in condition.

## Multi Collar Bearing

Let $n$ = no. of collars

$$P_{ind} \leq P_{per}$$

$$\frac{W_{each}}{\pi(R_o^2 - R_i^2)} \leq P_{per}$$

o. $\frac{W}{n(R_o^2-R_i^2)} \le P_{per}$

$\boxed{n = \frac{W_{each}}{W}}$

W = total load on the shaft

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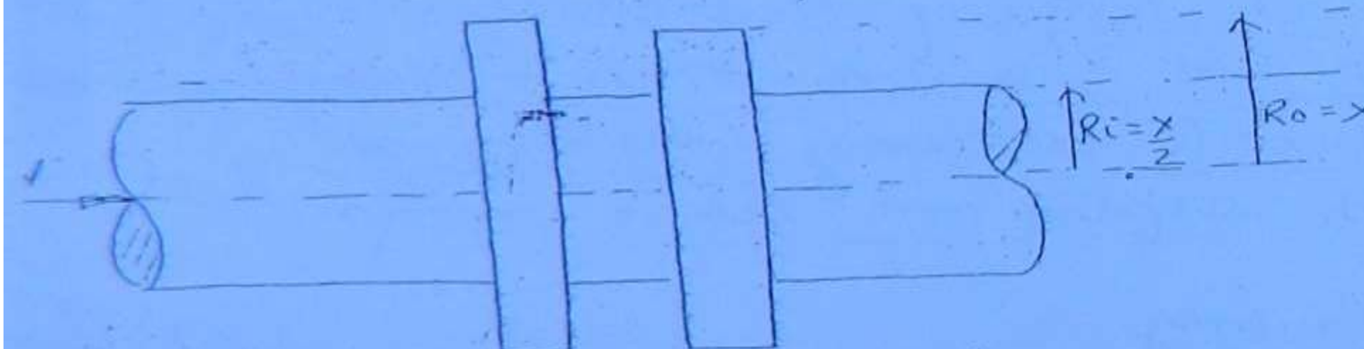
$\therefore n \ge$ — No.

s no. of collars increases, Load comming on each collar decreases and pressure induced decreases which leads to the safety against the failure of the clutch or collar.


$(T_f)_{MCB} = n\,(T_f)_{SCB}$

$= n \times \frac{2}{3}(\mu W)_{each}\left[\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right]$

$n = \frac{W}{W_{each}}$

$(T_f)_{MCB} = \frac{2}{3}\mu W\left[\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right]$

$(T_f)_{MCB} = $ [illegible]

and hence frictional Torque is independent

⇒ For a given load and given dimensions of the collars The frictional Torque in single collar bearing and multi collar bearings remains same ie, (frictional torque is independent of no. of collars) but the pressure induced at each collar in a multi collar bearing is less than the pressure induced at the collar of a single collar bearing.

CONICAL COLLAR BEARING (CCB) 155

$$P_{ind} = \frac{W}{\pi [R_o^2 - R_i^2]}$$

$$(T_f)_{CCB} = \frac{1}{\sin\alpha} (T_f)_{SCB}$$

There is no effect of cone angle on the pressure induced or load carrying capacity of the bearing but frictional torque or power loss is inversely proportional to $\sin\alpha$ (where $\alpha$ is the semi cone angle).

$\alpha = 90° \Rightarrow$ CCB becomes SCB

as $\alpha \uparrow \Rightarrow \sin\alpha \uparrow \Rightarrow T_f \downarrow \Rightarrow P_{Loss} \downarrow$ ] → CCB

$\alpha \downarrow \Rightarrow \sin\alpha \downarrow \Rightarrow T_f \uparrow \Rightarrow P_T \uparrow$ ] Cone clutch

In Conical collar bearing

$2\alpha = 120°$ to $160°$

[illegible]

In case of cone clutch

$\boxed{\alpha = 7\tfrac{1}{2} \text{ to } 15^\circ}$

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so, as to avoid self engagement of clutch

when the intensity of pressure is uniform in a flat pivot bearing of radius 'r' The friction force is assumed to act at

(i) $r$ (ii) $\frac{r}{2}$ (iii) $\frac{2}{3} r$ (iv) $\frac{r}{3}$

Which of the following statement valid for a multi collar thrust bearing carrying an axial thrust of W units.

1. friction moment is independent of no of collars.
2. coefficient of friction of bearing surface is effected by the no of collars
3. Intensity of pressure is effected by number of collars

(a) 1 and 2 (b) 2 and 3

(c) 1 and 3 (d) 1, 2, and 3

A multi collar thrust bearing having 300 mm and 400 mm as inner and outer diameters respectively determine, no of collars reqd for the bearing if permissible pressure is 7 kN/mm² and W is 1750 MN ?

Sol$^n$ $n = \dfrac{W}{W_{each}}$

$= \dfrac{1750 \times 10^6}{p \times \pi \cdot (R_o^2 - R_i^2)}$ (157)

$= \dfrac{1750 \times 10^6}{7000 \times \pi \times (200^2 - 150^2)}$

$= 4.47$

$\therefore n = 5$

Q: Repeat the above for thickness of collar if permissible shear stress is 60 MPa ?

Sol$^n$ $\dfrac{W_{each}}{n \cdot \pi D_i t} \leq \tau_{per}$

$\Rightarrow \dfrac{1750 \times 10^6}{5 \times \pi \times 300 \times t} \leq 60$

$\Rightarrow t \geq 6.8$ mm

$\therefore t = 7$ mm

JOURNAL BEARINGS

or (Radial bearings)

They are used to support a shaft which is subjected ~~\to~~ to radial loads (ie, loads acting perpendicular shaft axis)

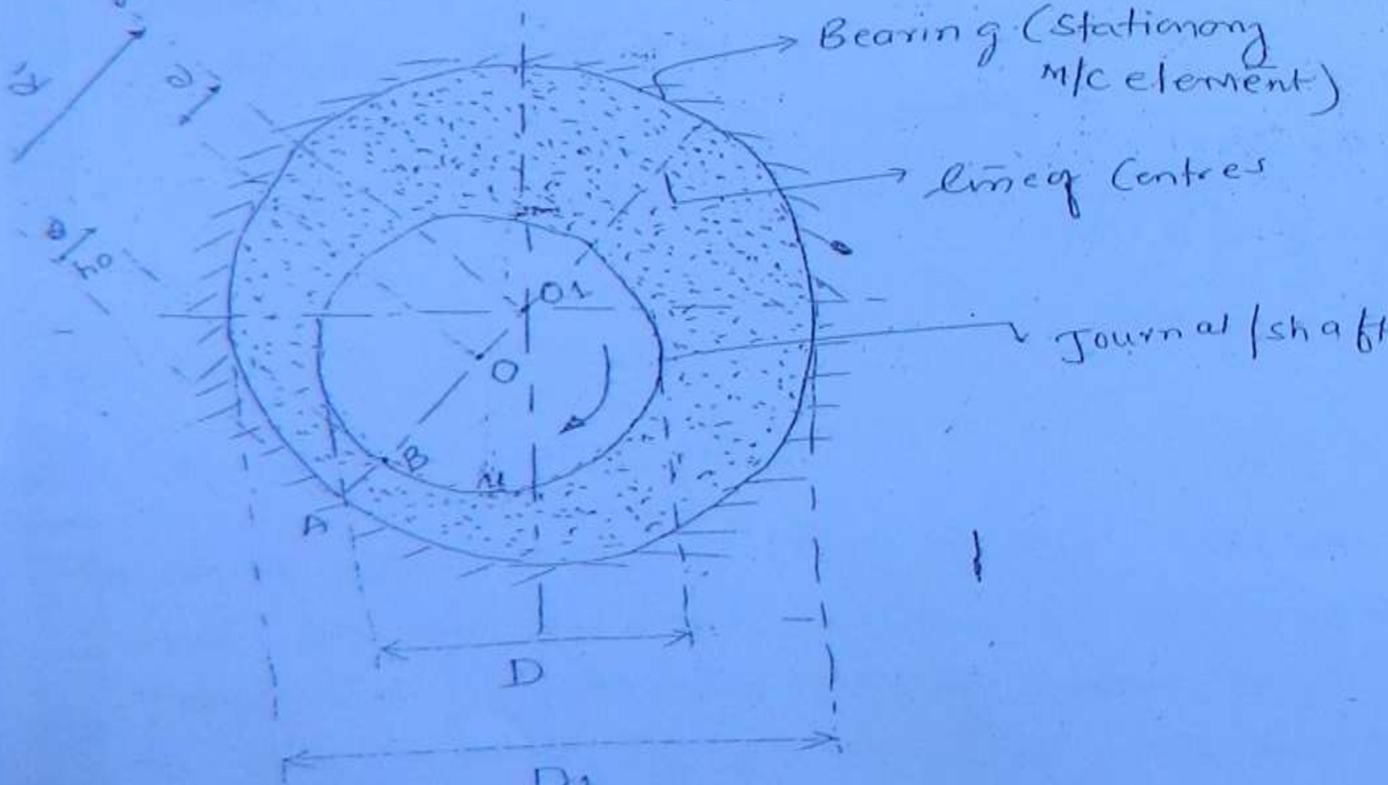
eg weight of pulley on horizontal shaft,

belt tensions, weight of gears. (Fr & Ft)

Terminology used in Journal bearing 156

Journal bearing is defined as a sliding contact radial bearing which is operating with hydrodynamic Lubrication.

They are suitable for high speed condition




position of Journal/shaft in Journal bearing at

$D_1$ = diameter of bearing

$D$ = diameter of Journal/shaft

$h_0$ = minimum film of thickness

$e$ = eccentricity

$C$ = diametral clearence

$$\boxed{C = D_1 - D}$$

$\Rightarrow D_1 = D + C$

$C_1$ = Radial clearence

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$C_1 = R_1 - R = \frac{C}{2}$

$$\boxed{C = 2C_1}$$

$\Rightarrow$ $\boxed{\frac{C}{D} = \text{diameteral clearence ratio}}$

If $C \downarrow \Rightarrow \frac{C}{D} \downarrow \Rightarrow$ heat generated or power loss increases (undesirable)

and load carrying capacity Increases (desirable)

If $C \uparrow \Rightarrow \frac{C}{D} \uparrow \Rightarrow$ heat generated or power loss decreases (desirable)

and load carrying capacity decreases (undesirable)

For a good bearing optimum value of $\frac{C}{D}$ is

$\frac{C}{D}$ is 0.001 to 0.002

$\Rightarrow$ $\frac{L}{D_1}$ ratio

L = length of bearing or Length of Journal

$D_1$ = diameter of bearing

If $\frac{L}{D_1} = 1 \Rightarrow$ Square bearing

$\frac{L}{D_1} < 1 \Rightarrow$ Short bearing

$\frac{L}{D_1} > 1 \Rightarrow$ Long bearing

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$\Rightarrow L \downarrow \Rightarrow \frac{L}{D_1} \downarrow \Rightarrow W \downarrow$ (undesirable)

$\Rightarrow$ and side leakage of lubricant is more

$\Rightarrow$ and effective lubrication decreases

$\Rightarrow$ and heat generated or power loss increases

$\Rightarrow L \uparrow \Rightarrow \frac{L}{D_1} \uparrow \Rightarrow W \uparrow$

$\Rightarrow$ Side leakage is less

$\Rightarrow$ effective lubrication increases

$\Rightarrow$ Heat generated or power loss decreases

Here $\frac{L}{D_1} > 1 \Rightarrow \boxed{\frac{L}{D_1} = 1 \text{ to } 2}$ *

shafts are available upto 6 to 7 m.

when $\frac{L}{D_1}$ is too Large there is a alignment problems for shaft and bearing

## bearing pressure ($P_p$)

$$\boxed{P_b = \frac{W}{LD_1} \le P_{permissible}}$$

Where LD is the projected area

$\therefore L \ge$ __ mm can be calculated.

W = Load comming on shaft

$W \le P_{per} \times L \times D$ (161)

→ Maximum load carrying capacity of Journar bearing

$$\therefore \boxed{W = P_{per} \times L \times D}$$

so on increasing Length, Load carrying capacity increases

## Eccentricity (e)

$R_1 = e + R + h_0$

$e = R_1 - R - h_0$

$e = c_1 - h_0$

$$\boxed{e = \frac{c}{2} - h_0}$$ ** *

entricity ratio or attitude of bearing ($\varepsilon$)

$$\varepsilon = \frac{\text{eccentricity}}{\text{Radial clearance}}$$

$$\varepsilon = \frac{2e}{c}$$

$$\varepsilon = \frac{2}{c}\left[\frac{c}{2} - h_o\right]$$

$$\boxed{\varepsilon = 1 - \frac{2h_o}{c}}$$ ***

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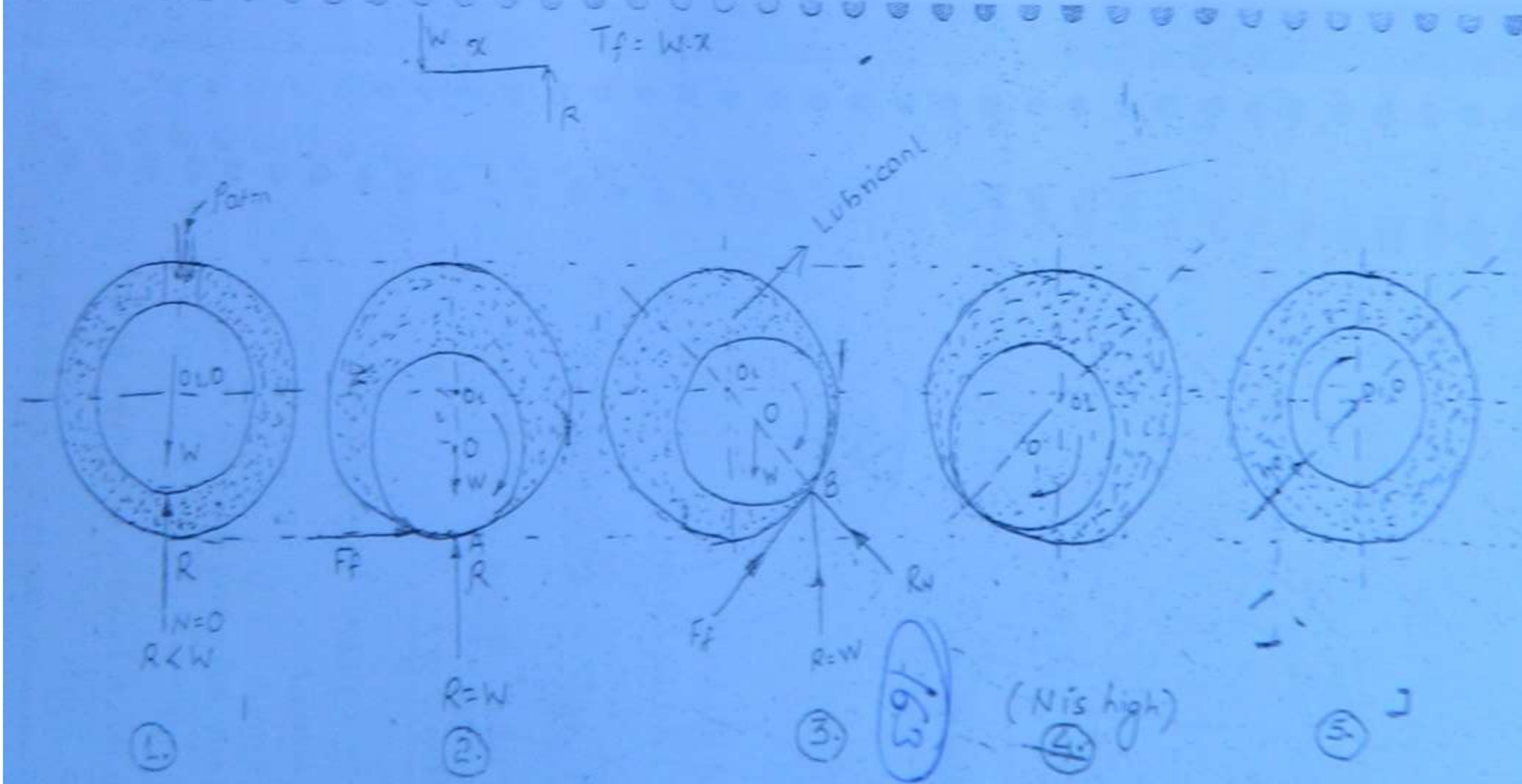
HYDRODYNAMIC LUBRICATION


$T_f = W \cdot x = 10\ N \cdot m$ [CCW]

$T_{apply} = \frac{P \times 60}{2\pi N} = 100\ Nm$ [CW]

→ and hence 10 N·m power loss occurs in utilising in overcomming frictional torque

1. N ↑ P ↑ → film or layer (sticky) and hence wedging action takes place and due to this pressure increases

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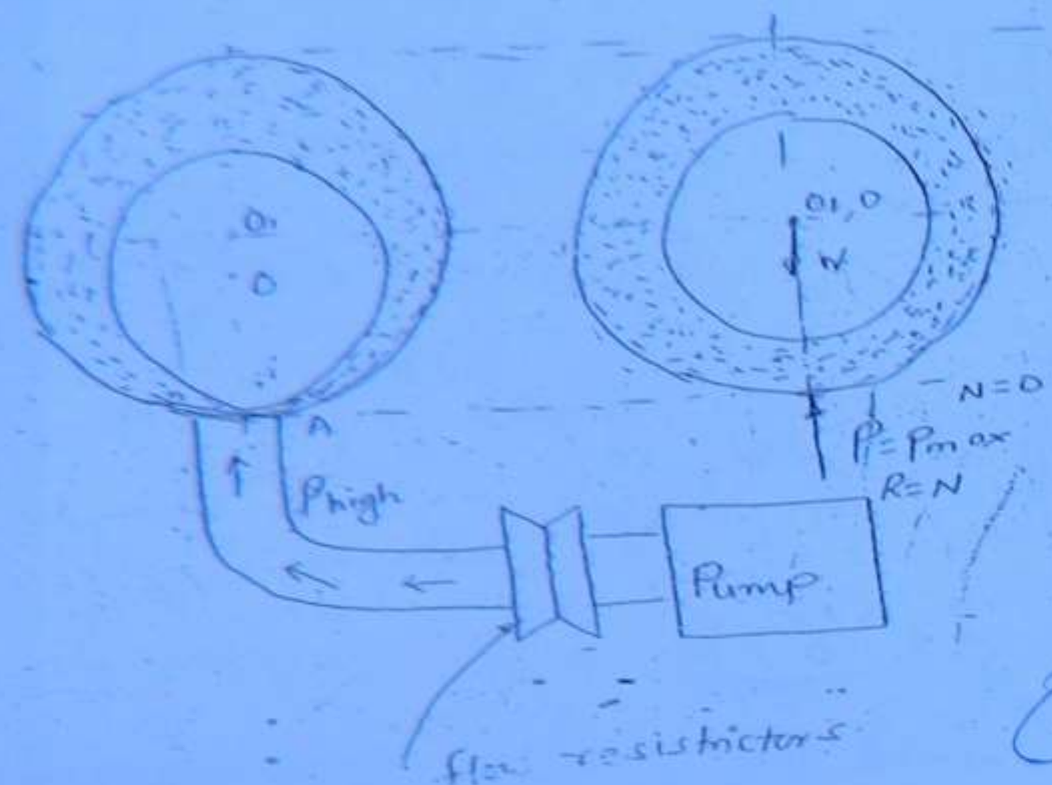
<u>hydrodynamic lubrication</u>

# Hydrostatic Lubrication

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| Hydrodynamic Lubrication | Hydrostatic Lubrication |
| --- | --- |
| Wedging action | 1) Ext device like pump |
| High speed application | 2) at any speed (ie, evenat Sationary) |
| metal to metal ct. can be avoided at high speed | 3) No metal to metal contact |
| high starting torque is required | 4) Low starting torque ie, min |
| cost is less | 5) Initial and maitenence cost is high |
| | 6) ... shaft is concentric |

6. They are used when shaft are subjected to less load at the stationary condition.

7. IC engine – crank shaft

6. shaft is subjected to high loads at stationary conditions.

7. Vertical turbogenerators and centrifuges and Ball mills

## FRICTION CIRCLE

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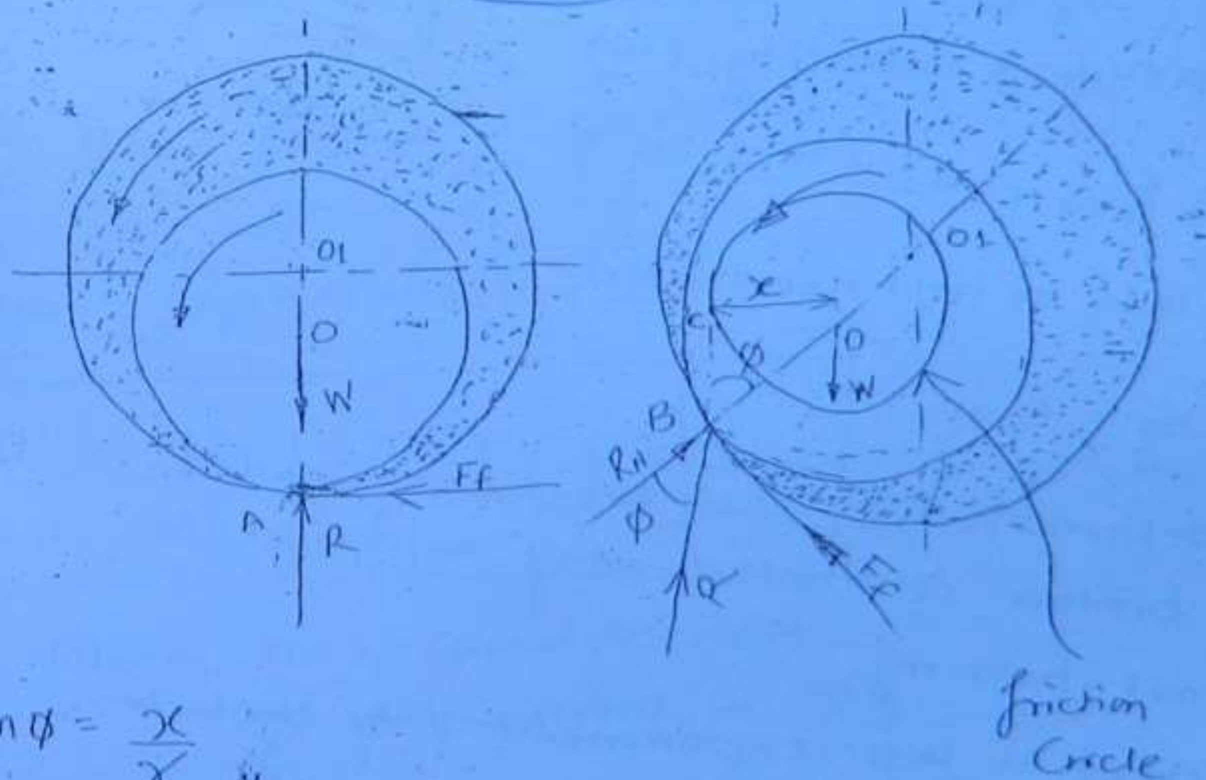


$$\sin\phi = \frac{x}{r}$$

$$x = r\sin\phi$$

$$x \simeq r\tan\phi \quad [\because \phi \text{ is small} \Rightarrow \sin\phi \approx \tan\phi]$$

$$x \simeq \mu r \quad [\because \mu = \tan\phi]$$

$$T_f = W\mu r$$

$$T_f = \mu W r$$

then shaft is in stationary condition the resultant force is inline with the line of action of the load acting on the shaft but when shaft is running condition, due to frictional force, resultant force get displaced from the line of action of load acting on the shaft by the distance which is equal to friction circle radius ( $oc = \mu r$ ).

Design criteria used in Journal bearing

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- Load carrying capacity (W)

$$W = P_{per} \times L \times D$$

- Power loss or heat generated (Hg)

$$P \text{ or } H_g = \mu W V$$

Now to determine $\mu$ Two, brothers called Mc-Kees brother conducted no. of experiments on Journal bearing and based on their experiment they conclude that $\mu$ is a function of

(1) $\mu \propto f\left[\left(\frac{Zn}{P}\right), \left(\frac{D}{C}\right) \& \frac{L}{D}\right]$

value of $\frac{C}{D}$ range from 500 to 1000

Range of $\frac{L}{D} = 1$ to $2$

<u>bearing characteristic Number</u> $= \frac{Z \cdot n}{P}$

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where $Z$ = absolute viscosity of Lubricant at operating temp. as $(t_o)$. in pa.s or $\frac{kg}{m \cdot s}$ or $N s/m^2$

$$\boxed{1 \text{ c.p} = 0.001 \text{ pa.s}}$$

$N$ = speed of Journal in RPS

$P$ = bearing pressure, $= \frac{W}{LD}$ in pa

$$BNO = \frac{pa \cdot \cancel{s}}{pa} \times \frac{1}{s} \quad (\text{No unit}).$$

They gave expression for '$\mu$' as

$$\boxed{\mu = \frac{33}{10^8}\left[\left(\frac{Zn'}{P'}\right)\left(\frac{D}{C}\right)\right] + K}$$

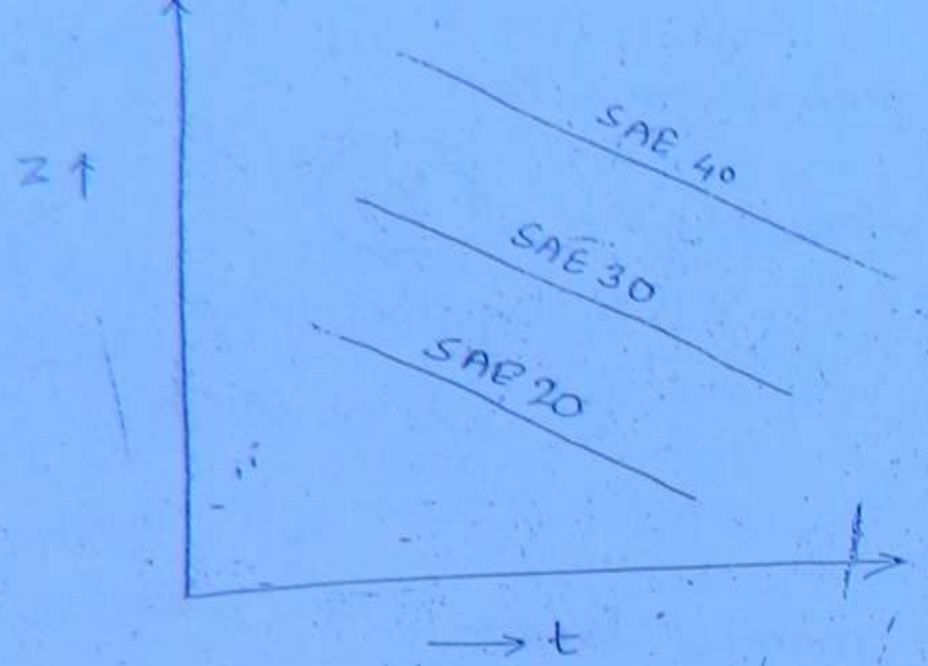
where $n'$ = speed in RPM

$P'$ = pressure in MPa

$Z$ = pa.s or $\frac{kg}{m \cdot s}$ or $NS/m^2$

$\mu$ = coefficient of friction

$K$ is constant depend on $\frac{L}{D}$ ratio.

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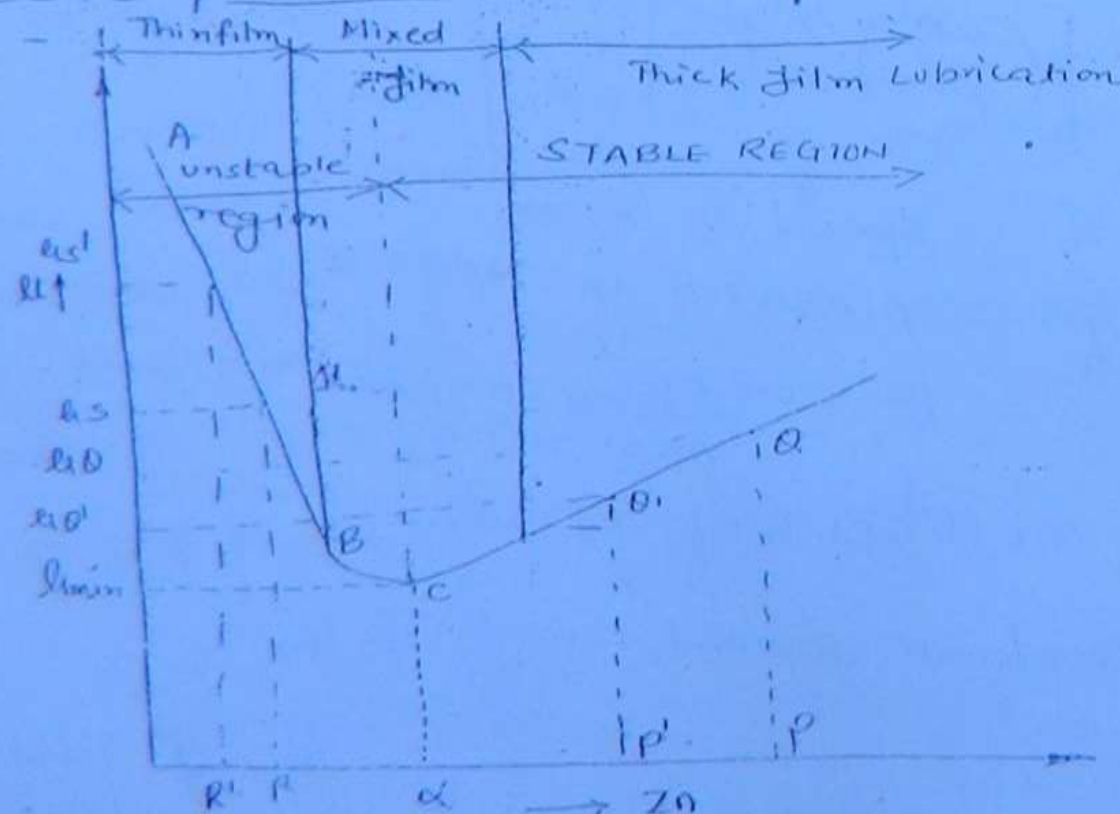
$\Rightarrow K = 0.002 \Rightarrow$ if $\frac{L}{D}$ has value

$$0.75 \le \frac{L}{D} \le 2.8$$

$\Rightarrow K = 0.003 \Rightarrow$ if $\frac{L}{D} > 2.8$

Effect of $\frac{Zn}{P}$ on coefficient of friction ($\mu$)



## Bearing modulus ($\alpha$)

It is the value of bearing characteristic number corresponding to minimum coefficient of friction

always $\frac{Zn}{P}$ taken greater than $\alpha$.

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as $T \uparrow$, $Z \downarrow$

and $\frac{Zn}{P} \downarrow$ and due to this $\mu \downarrow$

and hence heat generated is $\downarrow$

and due to this $T \uparrow$, $Z \uparrow$ and $\frac{Zn}{P} \uparrow$

Also

$W \uparrow$, $\frac{Zn}{P} \downarrow$, $\mu \downarrow$, $H_g \downarrow$, $T \downarrow$, $Z \uparrow$ and

finally $\frac{Zn}{P} \uparrow$ and stable condition is achieved.

For unstable region

$T \uparrow$, $Z \downarrow \Rightarrow \frac{Zn}{P} \downarrow$, $\mu \uparrow \Rightarrow H_g \uparrow$, $T \uparrow$, $Z \downarrow$

and due to this $\frac{Zn}{P} \downarrow$

generally $\frac{Zn}{P} \geq 3\alpha$ (steady condition)

Sometime $\frac{Zn}{P} \geq 15\alpha$ (under highly fluctuating Load condition)

<u>iscosity index (VI)</u>

It is a measure of change of viscosity with change in temperature

$$VI = \frac{dZ}{dt} = \frac{Z_2 - Z_1}{T_2 - T_1}$$

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* <u>Somerfield Number (S)</u>

$$\boxed{S = \left(\frac{Zn}{p}\right)\left(\frac{D}{C}\right)^2}$$

n is in rps

p is pa or N/m2

Z is in pa·s

Somerfield number remains constant for given journal bearing (i.e., for a given L & D) hence it is use to correlate the working condition of different machine which are operating with the same journal bearing.

M/c 1

$L_1 = 500 mm$

$D_1 = 250 mm$

$W_1 = 10 kN$

$N_1 = 1000 rpm$

(17)

Now New Machine 2

$L_2 = 500 mm$

$D_2 = 250 mm$

$W_2 = 20 kN$

$N_2 = ?$

on equating 'S'

ie, $S_1 = S_2$

$$\left(\frac{Z_1 n_1}{P_1}\right)\left(\frac{\cancel{D_1}}{\cancel{C_1}}\right)^2 = \left(\frac{Z_2 \cdot n_2}{P_2}\right)\left(\frac{\cancel{D_2}}{\cancel{C_2}}\right)^2$$

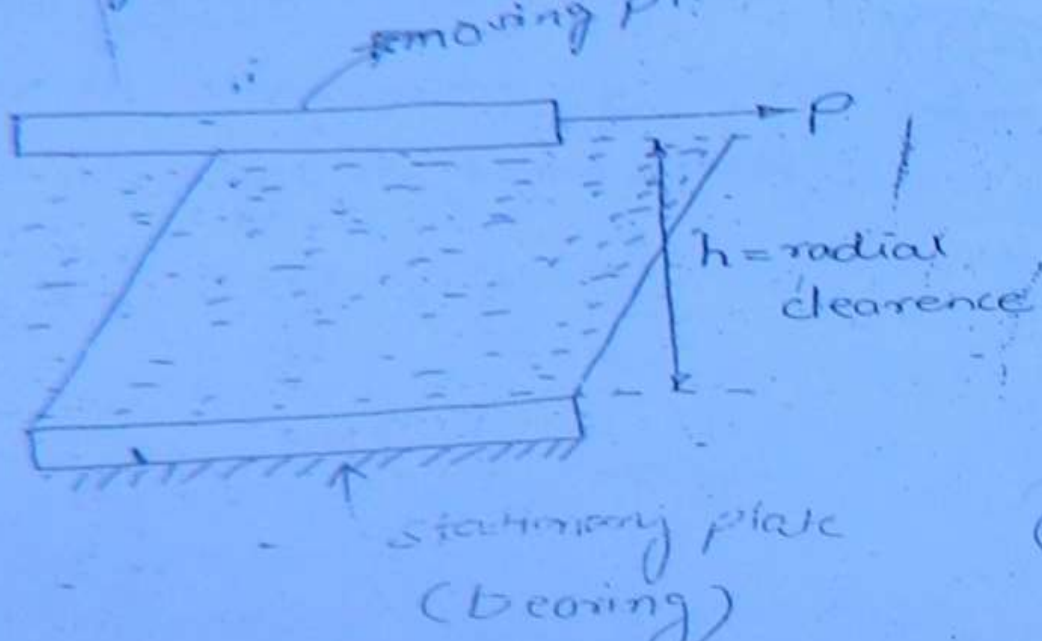
$$\frac{n_1}{p_1} = \frac{n_2}{P_2}$$ {Same Lubrication)

$$\frac{1000}{\cancel{P_1}} = \frac{n_2}{2P_1}$$

$\therefore n_2 = 2000$ RPM

$P_{loss} = T_f \cdot \omega$ , $T_f = \mu F_r \cdot r = \mu \cdot W \cdot r$

xpression for frictional Torque in terms of iscosity ($T_f$)

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according to Newton's Law of viscosity

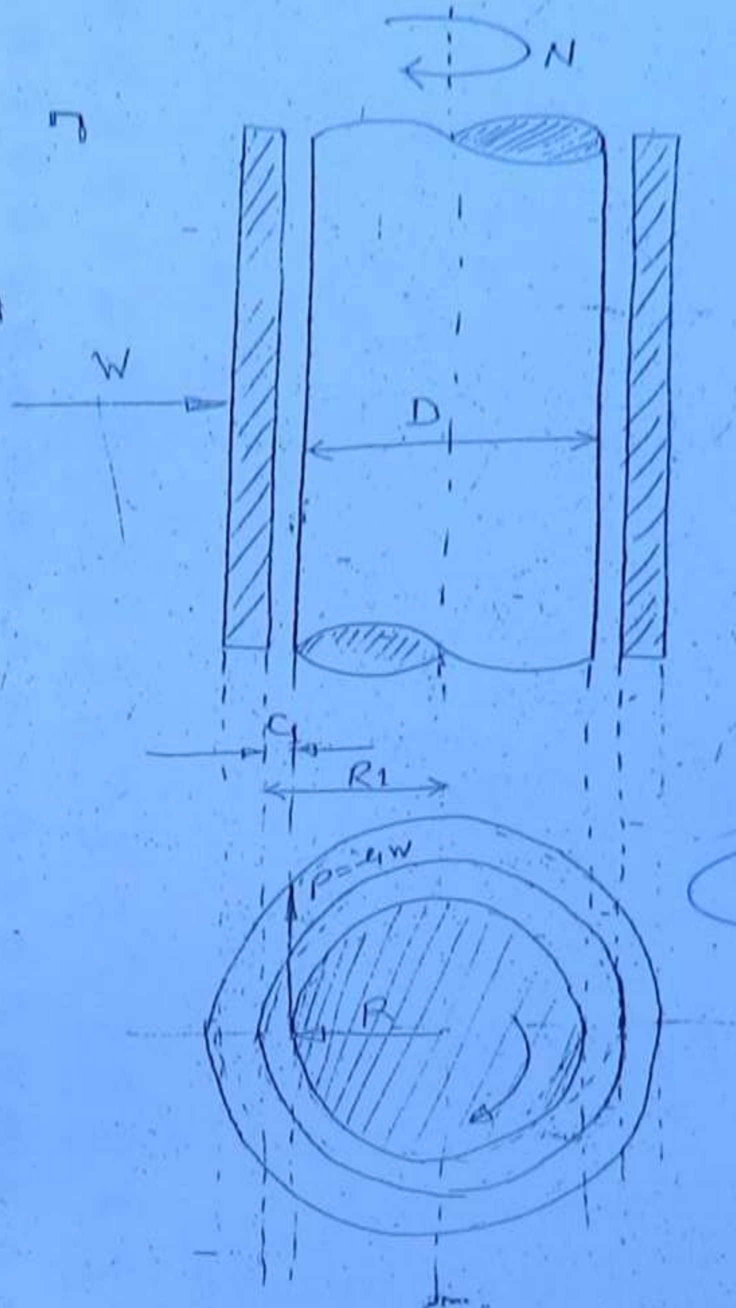
$$\tau \propto \frac{v}{h}$$

$$\tau = \frac{Zv}{h} \longrightarrow (1)$$

$$\tau = \frac{P}{A} \longrightarrow (2)$$

$$\therefore \frac{P}{A} = \frac{Zv}{h}$$

Now, $\boxed{P = \frac{Zv}{h} \times A}$

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$$P = Z \times \frac{\pi D N'}{60} \times \frac{\pi D L}{C_1}$$

$$= \frac{Z \pi^2 D^2 L N' \times 2}{60 \times C}$$

$$T_f = \mu W R$$

$$= \frac{Z \pi^2 D^2 L \times N' \times 2}{60 \times C} \times \frac{D}{\cancel{2}}$$

$$\boxed{T_f = \frac{Z\pi^2 D^3 L n'}{60 \times C}} \longrightarrow (I)$$

Now $T_f = \mu W \frac{D}{2}$ — (II)

equating 1 and 2.

$$\mu \cdot W \cdot \frac{D}{2} = \frac{Z\pi^2 D^3 L n'}{60 \times C}$$

$$\frac{\mu \times p \times L \times D^2}{2} = \frac{Z \times \pi^2 D^3 \times L \times n'}{60 \times C}$$ (174)

$$\mu = 2\left(\frac{Zn'}{p}\right)\frac{D\pi^2}{60C} = \frac{\pi D^2}{30C}\left(\frac{Zn'}{p}\right)$$

∴ petroff's equation hence is

$$\boxed{\mu = \frac{\pi^2}{30}\left(\frac{Zn'}{p}\right)\left(\frac{D}{C}\right)}$$ * remember directly

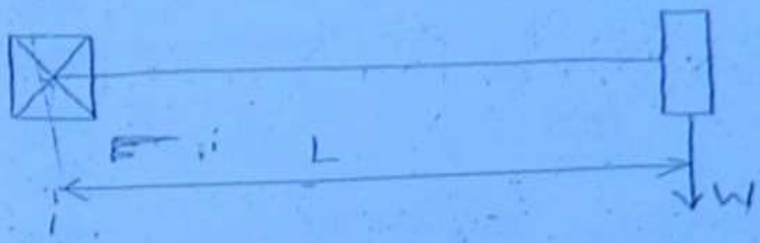
Design procedure used in Journal bearing

1) Shaft diameter or journal diameter [D] by MSST

$$T_e = \sqrt{(K_b M)^2 + (K_t T)^2} = \frac{\pi}{16} D^3 \tau_s$$

$$T = \frac{P \times 60}{2\pi N} \times 10^6 = \_\_\_ \text{ Nmm}$$

P is in kW




$M = W \cdot L$

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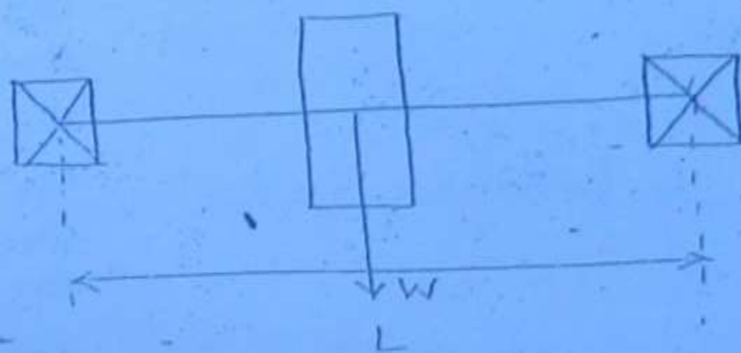


$M = \frac{WL}{4}$

by MDET

$$M_e = \sqrt{(K_b \cdot M)^2 + \frac{3}{4}(K_t \cdot T)^2} = \frac{\pi}{32} D^3 (\sigma_b \text{ or } \sigma_t)$$

$\therefore D = ?$

(2) Bearing Diameter $(D_1)$

$$D_1 = D + C$$

$$\frac{C}{D} = 0.001 \text{ to } 0.002$$

3) Length of bearing

$$P_{ind} \le P_{per}$$

$$\frac{W}{LD} \le P_{per}$$

$$\therefore L \ge \_\_\_ mm$$

calculated $\frac{L}{D}$ should be Less or equal to

$\left(\frac{L}{D}\right)$given

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(4) Power Loss or (Hg).

$$P_L \text{ or } H_g = \mu W v$$

$$= T_f \times \frac{\omega}{2}$$

Now according to Mc-kee's equation

$$\mu = \frac{33}{10^8}\left[\left(\frac{Zn'}{P'}\right)\left(\frac{D}{c}\right)\right] + K$$

$n' \stackrel{\text{is}}{=}$ in rpm

$p' =$ Mpa

$K \rightsquigarrow \frac{L}{D}$ ratio

$$v = \frac{\pi D N_{rpm}}{60 \times 1000} = \_\_\_ m/s$$

$\therefore P_{Loss} = \mu \cdot W \cdot V^{\to m/s} = \_\_ \text{ watts}$

↳ Newton

$\therefore P_{Loss} = T_f \cdot \omega'$

$$\boxed{T_f = \frac{2\pi^2 D^3 L n'}{60 \cdot C}}$$

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$$\omega' = \frac{2\pi n'}{60} = \ldots r/s$$

5) <u>Heat dissipated</u> (Hd)

$$Hd = C_d \cdot (t_b - t_a) \times L^{\to m} \times D^{\to m}$$

$= \_\_\_$ watts

$C_d$ = heat dissipation coefficient

$C_d \Rightarrow$ unit is $W/m^2 \, °C$

$$\boxed{\Delta t = t_b - t_a = \frac{1}{2}\left[t_o - t_a\right]}$$

$t_o$ = operating temperature

$t_a$ = atmospheric temperature

$\therefore$ Hd can be calculated in watts

$t_b$ = temperature of bearing surface

nd Method "Lasche's equations"

$$\uparrow H_d = \frac{(\Delta t + 18)^2 \, L \cdot D}{K \downarrow}$$

K = heat dissipation constant

K = 0.273 ⇒ well ventilated bearing

K = 0.484 ⇒ still air conditioning

∴ Hd can be calculated.

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) check for artificial cooling is required or not

(i) If Hg = Hd ⇒ no artificial cooling is required

(ii) If Hg > Hd ⇒ artificial cooling is required.

) volume flow rate of coolant

Heat to be carried away by coolant (Hc) is

$$H_c = H_g - H_d = \text{---} \text{ watts}$$

$$H_c = m \cdot s \cdot (t_o - t_i)$$

m: kg/s

s: specific heat

to and ti are inlet and outlet temp of cooling medium.

$\therefore$ $m = \_\_$ kg/s

$S = 1840 - 2100$ J/kg/°C

$V = ?$ ; by $\rho = \frac{m}{V}$

$\therefore$ $V = \frac{?}{\phantom{x}}$ m/s.

$=$ Lit/hr

1 litre $=$ 1000 cc

(179)

IES

Q: a full Journal bearing having clearence to radius ratio of 1/100 using a Lubricant with $\mu$ is equal to $28 \times 10^{-3}$ pa-s, supports the Journal running at 2400 RPM if bearing pressure 1.4 mpa, the sommerfield Number is ?

Soln $\frac{\text{clearence}}{\text{radius}} = \frac{1}{100} = \frac{c_1}{r}$

$Z = \mu = 28 \times 10^{-3}$ pa-s

$n' = 2400$ rpm

$p' = 1.4$ mpa

$$S = \left(\frac{Zn}{P}\right)\left(\frac{D}{C}\right)^2$$

$$= \left(\frac{28\times10^{-3}}{1.4\times10^{6}} \times \frac{2400}{60}\right)(100)^2$$

$$\frac{C_1}{R} = \frac{1}{100}$$

$$\frac{C/2}{D/2} = \frac{1}{100}$$

$$\therefore \frac{D}{C} = 100$$

$$S = 0.008$$

Q: A journal bearing of diameter 50 mm and the length 50 mm, operating at =20 rps, carries a load of 2KN, the lubricant used has a viscosity of 20 mpa-s, the radial clearence is 50 µm ($1\,\mu m = 10^{-6} m$) The sommerfield number for this bearing is ?

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Ans

$D = 50$ mm

$L = 50$ mm

$n = 20$

$W = 2$ KN

$Z = 200 \times 10^{-3}$ pa.s

$C_1 = 50 \times 10^{-6}$ m

$S = ?$

$$S = \left(\frac{Zn}{P}\right)\left(\frac{D}{C}\right)^2$$

$$P = \frac{W}{LD} = \frac{2\times10^{3}}{50\times10^{-3}\times50\times10^{-3}}$$

$$\therefore S = \left[ \frac{20\times10^{-3}\times 20}{\frac{2000}{0.05\times0.05}} \right] \left[ \frac{50}{2\times 50\times10^{-6}\times10^{3}} \right]^{2}$$

$$S = 0.125.$$

Q: a Journal bearing

$D = 40\,mm$

$L = 40\,mm$

$n = 20\ rad/s$

$Z = 20\ mpa\text{-}s$ (181)

$C = 0.02\,mm$

determine Loss of torque due to viscosity of the Lubrication.

Soln

$$T_f = \frac{Z\pi^2 D^3 n' L}{60\,C}$$

$$= \frac{20\times10^{-3}\times\pi^2\times(40\times10^{-3})^3\times 40\times10^{-3}}{60\times 0.02\times10^{-3}}$$

$$T_f = 0.08$$

$$V = \frac{\pi D n'}{60} = \frac{D}{2}\omega'$$

$n'$ can be calculated.

a Journal bearing of 50 mm diameter
nd 80 mm long has a bearing pressure
f 6 Mpa is used to support a journal
unning at 1000 RPM. The bearing is
ubricated with oil whose absolute viscosity
t the operating temp. of 75°C may be
aken as 0.015 kg/ms, room temperature
25°C (ta), $\left(\frac{D}{c} = 1000\right)$.

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determine the following

amount of artificial cooling required
) mass of coolant oil required. if specific
heat of the oil 1900 J/kg/°C and difference
of inlet and outlet temp of coolant oil is
25°C and heat dissipation coefficient
500 W/m²°C.

$\mu = 0.00283$

$H_g = \mu W V = 177.95\ W$

$V = 2.62\ m/s$

$H_d = 50\ W$

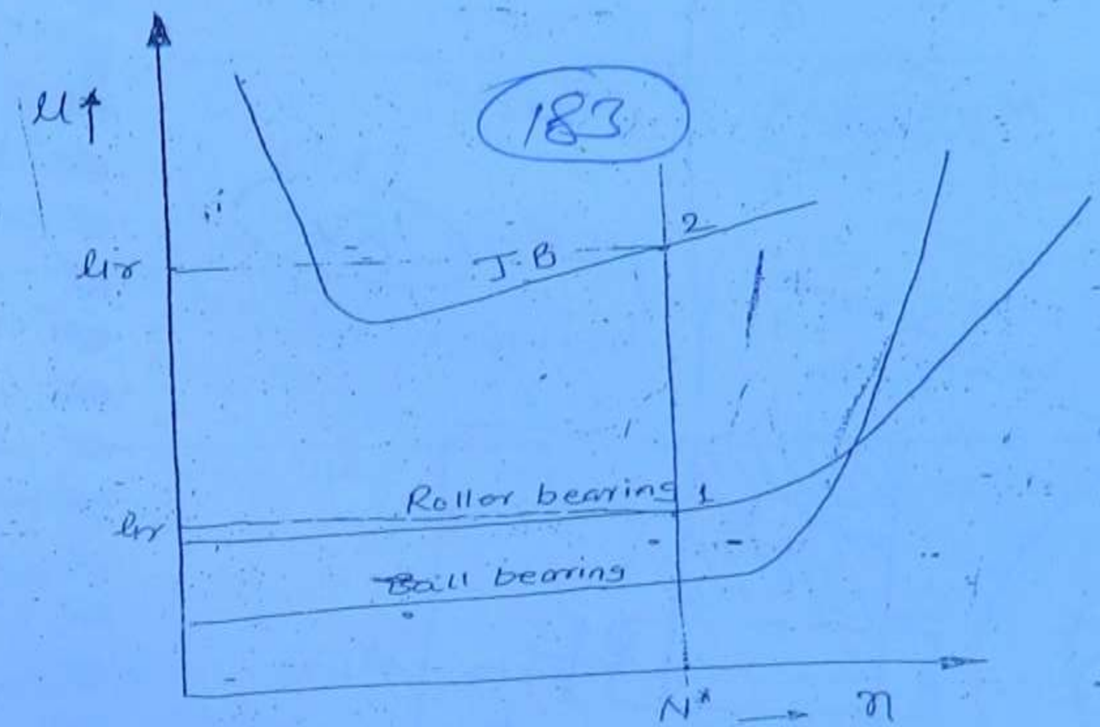
$H_c = H_g - H_d = 127.95\ W$

$m = 0.0067\ kg/s$

# Antifriction Bearing*

Or

# Rolling Contact Bearing* [RCB]




→ Used for Low or medium speed range.

$N^*$ ⇒ running condition (RPM)

| | parameters | Journal bearing | Antifriction bearing |
|---|---|---|---|
| 1. | Load | Fr | Fr and Fa |
| 2. | Speed | high speed application | Low and Medium speed application |
| 3. | Machines (application) | Used in m/c where there is continous application | intermittent application |

| parameters | Jornal bearing | Antifriction Bearing |
| --- | --- | --- |
| Cost | Less costly | more costly |
| Axial space | More | less |

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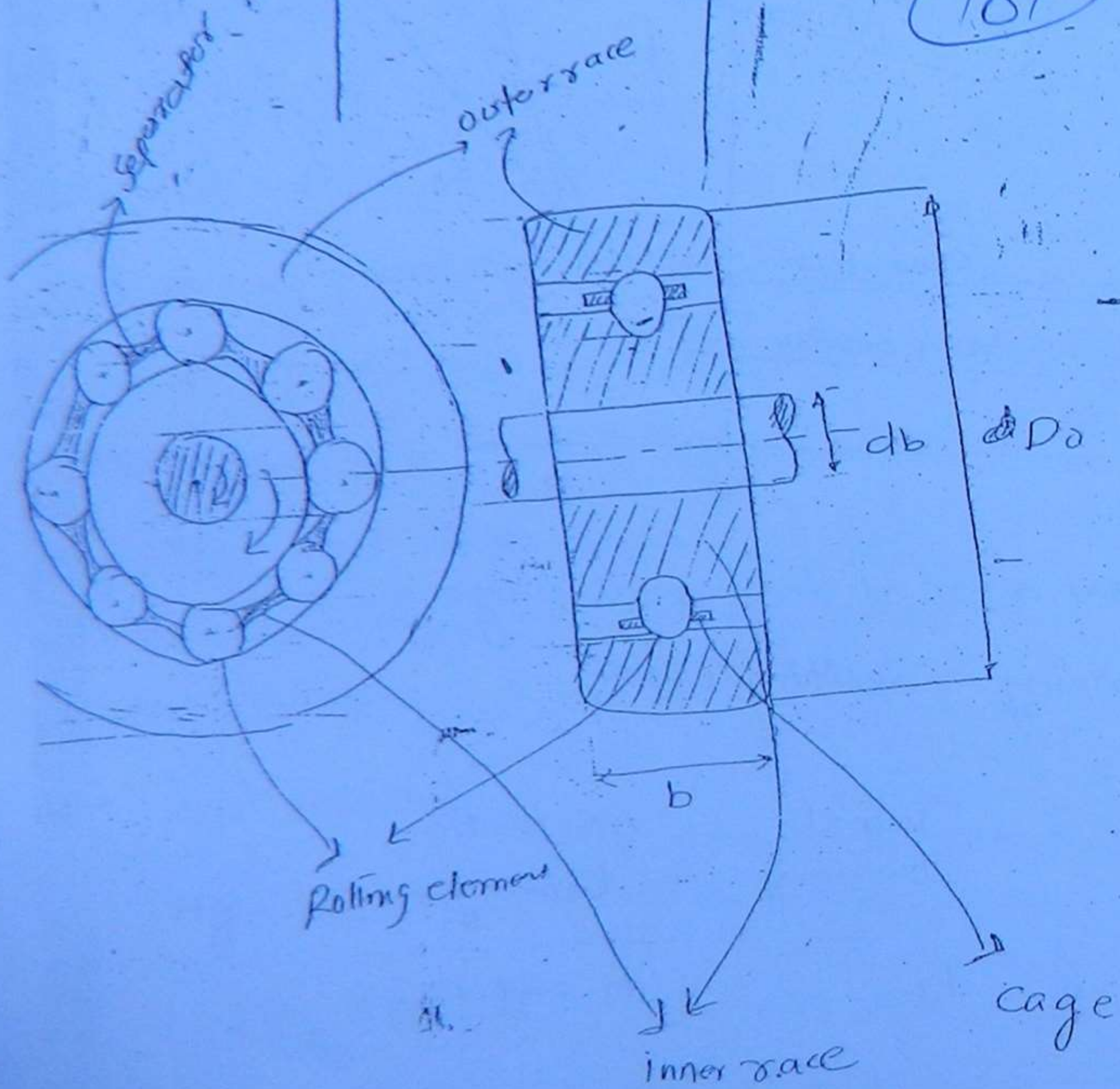

| parameter | Journal bearing | Antifriction bearing |
|---|---|---|
| damping capacity | Less | More |
| Radial space | Less | 185 more |
| precise alignment | not required | precise alignment |
| 8. starting torque | More | less |
| Lubrication | continuous lubrication is reqd | is not reqd |
| 3. Lubricant | Liquid | Semi solid |
| 1. Noise | Less | More |
| 2. Life | More | finite |

Antifriction designation

SKF 6308

BIS

40 BC 03

)- SKF 63(08)x5

Type of series

Type of Antifriction bearing

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6 → DGBB

(deep groove ball bearing).

(08)x5 → diameter of bore of diameter of shaft

every AFB is manufactured in 5 different series.

1. 6108 → 100 series → extra light series
2. 6208 → 200 series → Light series
3. 6308 → 300 series → medium series
4. 6408 → 400 series → heavy series
5. 6508 → 500 series → Extra heavy series

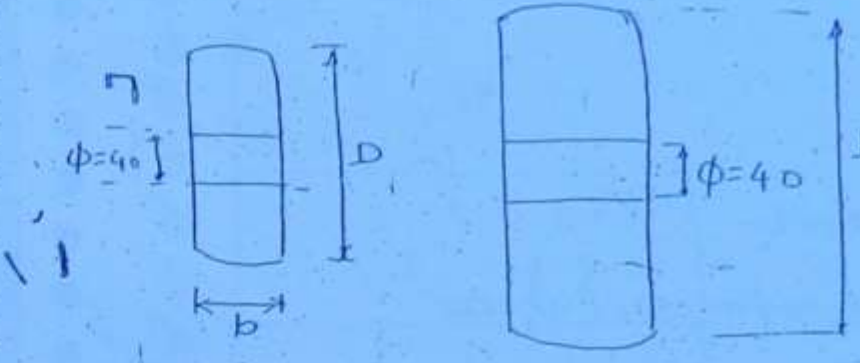
going from Top to bottom

→ Do and b increases

→ Cost increases

→ Load carrying capacity increases

1
2
3
4
5




6108

6208

(φ remains constant)

BIS ( Beaure of Indian standard)

Ex. BIS

40 BC 03

shaft diameter

Type of antifriction bearing

Series

03 ⇒ 300 Series

Terms used in the selection of series of an AFB.

① Equivalent load. [Pe]

Antifriction bearing manufacturing association (AFBMA) given Pe as

$$P_e = S\left[X V F_r + Y F_a\right]$$

S = Service or shock factor

$\Rightarrow S = 1 \Rightarrow$ steady loads

$1.5 \Rightarrow$ light shocks

$2 \Rightarrow$ Moderate shocks

$2.5 \Rightarrow$ heavy shocks/impact shocks

$3 \Rightarrow$ extra heavy shock

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$\Rightarrow V =$ race ~~propotion~~ rotation factor

$= 1 \Rightarrow$ inner race rotation

$1.2 \Rightarrow$ outer race rotation

$F_r =$ radial load

$F_a =$ axial load

$x =$ radial load factor

$y =$ Axial load factor

or Thrust ball bearing / Thrust roller bearing, withstand only axial load $[F_r = 0]$

$\therefore x = 0$ and $y = 1$

Cylindrical roller bearing

withstand only radial load $[F_a = 0]$

$\therefore x = 1, y = 0$

Pe formula is valid only if Fr and Fa are constant and N is constant

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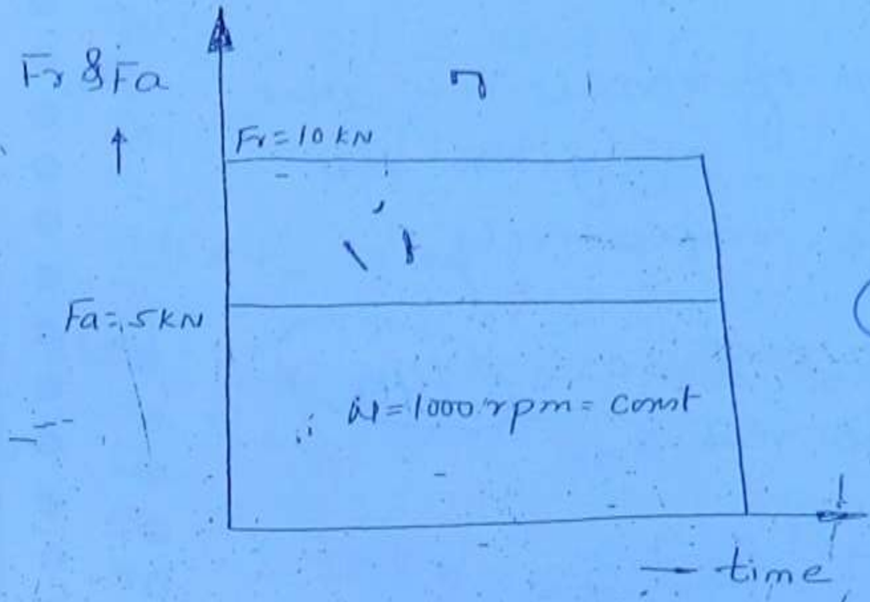
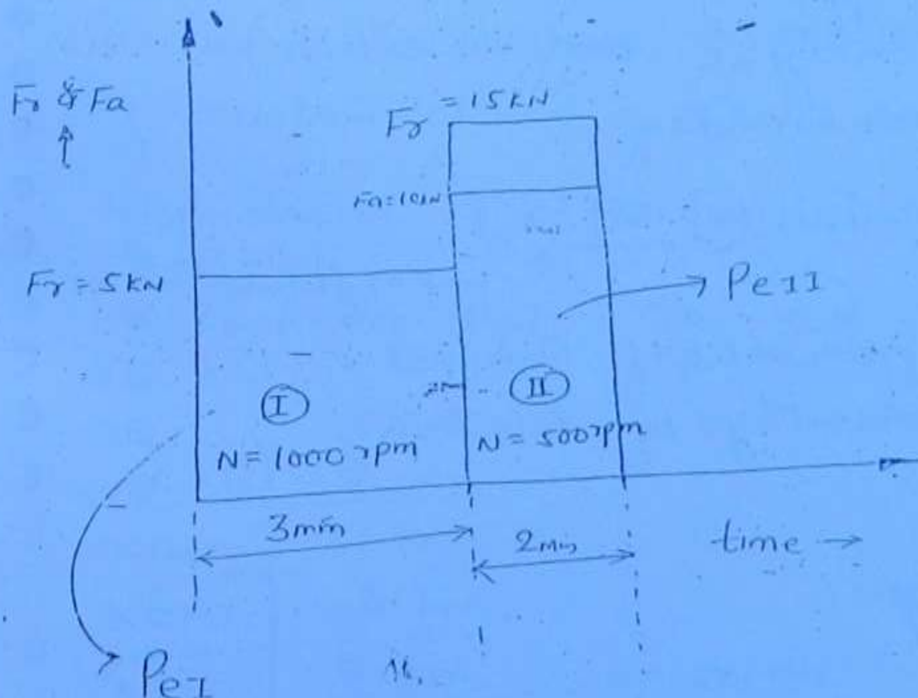



## for varying Load and RPM




Using Cubic Mean load formulae

$$\boxed{P_m = \sqrt[3]{\frac{P_{eI}^3 \cdot n_I + P_{eII}^3 \cdot n_{II}}{n_I + n_{II}}}}$$

$n_I$ and $n_{II}$ are no. of revolution ie, Shaft or where

$n_I$ and $n_{II}$ are the No. of revolutions that a bearing has undergone during the first stage and second stage respectively.

$\therefore n_I = 1000 \times 3 = 3000$ revs.

$n_{II} = 500 \times 2 = 1000$ revs.

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## Life of an Antifriction bearing

It is defined as the No. of revolution that a bearing has undergone before the evidence of first fatigue crack either in races or rolling elements.

Nominal or Rated Life [$L_{90}$] or [$L_{10}$]

$\uparrow$ prob. of failure.

probability of Survival or percentage of Reliability

SKF 6308

$\hookrightarrow$ 100 bearings.

$L_{90}$ = 100 million revolution.

90% of bearings Life $\geq$ 100 MR

10% of bearings life $\leq$ 100 MR

$$P_e = S[xVF_r + YF_a]$$

$$= 1[1 \times 1 \times 10 + 0]$$

$$P_e = 10 \text{ kN}$$

(191)

Now, $L_{90} = \frac{365 \times 24 \times 60 \times 1000}{10^6} = 525.6 \text{ mR}$

$$\therefore 525.6 = \left(\frac{C}{10}\right)^{10/3}$$

$$C = 65.49 \text{ kN}$$

Now see the catalogue to see which series can sustained C of 65.49 kN.

Cylindrical Roller bearing (x)

Last two digit $= \frac{100}{5} = 20$

| CRB | C |
|---|---|
| x120 | 10 kN |
| x220 | 25 kN |
| x320 | 50 kN |
| $\boxed{x420}$ | $\boxed{70 \text{ kN}}$ |
| x520 | 80 kN |

Selected

determine the condition for the load acting on a ball bearing if its life is to be halved? by how much life is to be increased?

Sol

$P_1 \Rightarrow$ Life is $L_1$

$P_2 \Rightarrow L_2 = \frac{L_1}{2}$

( $k = 3$ ) for ball bearing

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$$\frac{L_2}{L_1} = \frac{\left(\frac{\cancel{C_2}}{P_2}\right)^3}{\left(\frac{\cancel{C_1}}{P_1}\right)^3}$$

$$\frac{L_2}{L_1} = \left(\frac{P_1}{P_2}\right)^3$$

$$\left(\frac{L_1}{2L_1}\right)^{1/3} = \frac{P_1}{P_2}$$

$$P_2 = 2^{1/3} P_1$$

$$P_2 = 1.26 P_1$$

what is the life of a ball bearing if load acting on the ball bearing is halved?

Sol

$P_1 \Rightarrow$ Life is $L_1$

$\frac{P_1}{2} \Rightarrow$ Life is $L_2$

$$\boxed{\frac{L_2}{L_1} = \left(\frac{P_1}{P_2}\right)^3}$$

$$\frac{L_2}{L_1} = \left(\frac{P_1}{P_1/2}\right)^3$$

$$\therefore L_2 = 2^3 L_1$$

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$$L_2 = 8 L_1$$

$\therefore$ Life of the ball bearing is 8 times.

Q: What is the condition for the bearing if the life of the ball bearing is doubled when the load acting on the ball bearing is doubled?

Soln

$$\frac{L_2}{L_1} = \left(\frac{C_2/P_2}{C_1/P_1}\right)^3$$

$$= \left(\frac{C_2}{C_1}\right)^3 \left(\frac{P_1}{P_2}\right)^3$$

$$2 = \left(\frac{C_2}{C_1}\right)^3 \times \left(\frac{1}{2}\right)^3$$

$$C_2 = 2.52\, C_1$$

Select a new bearing whose

i.e., If the life to be doubled when load also becomes doubled a new bearing is to be selected whose dynamic capacity should is 2.52 times the original bearing dynamic capacity.

Q: SKF 6306 ball bearing with inner ring rotation have a 10 seconds works cycle as follows

| parameter | for 2 sec | for 8 seconds |
|---|---|---|
| $F_r$ | 3640 N | 2730 N |
| Fa(N) | 1820 N | 0 N |
| RPM | 900 | 1200 |
| Type of Load | Light shock | steady load |
| x and y values | $x = 0.56$<br>$y = 1.4$ | $x = 1$<br>$y = 0$ |

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$C = 22$ KN

find out expected average life in hrs (Long) hrs

Sln

| I Stage | II stage |
|---|---|
| $n_I = ?$<br>60 s — 900 rev<br>2 s — ?<br>$n_I = 30$ rev | 60 s → 1200 rev<br>8 s — ?<br>$n_{II} = 160$ rev |
| $P_{eI} = S[xVF_r + yF_a]$<br>$P_{eI} = 6.8796$ kN | $P_{eII} = 2.73$ KN |

$$P_m = \sqrt[3]{\frac{P_{eI}^3\, n_I + P_{eII}^3\, n_{II}}{n_I + n_{II}}}$$

$$P_m = \sqrt[3]{\frac{(6.8796)^3 \times 30 + (2.73)^3 \times 160}{30+160}}$$

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$$\therefore P_m = 4092.53 \text{ N}$$

$$L_{90} = \left(\frac{C}{P_m}\right)^3 = \left(\frac{22}{4092.53}\right)^3 = 155.34 \text{ million revolution}$$

$$L_{50} = 5 L_{90} = 776.7 \text{ million revolution}$$

$$n_I + n_{II} = 190 \text{ revs} \longrightarrow 10 \text{ s}$$

$$776.7 \times 10^6 \text{ rev}^n \longrightarrow ?$$

$$L_{50} = 11355.25 \text{ hrs}$$

ACBB = angular contact ball bearings

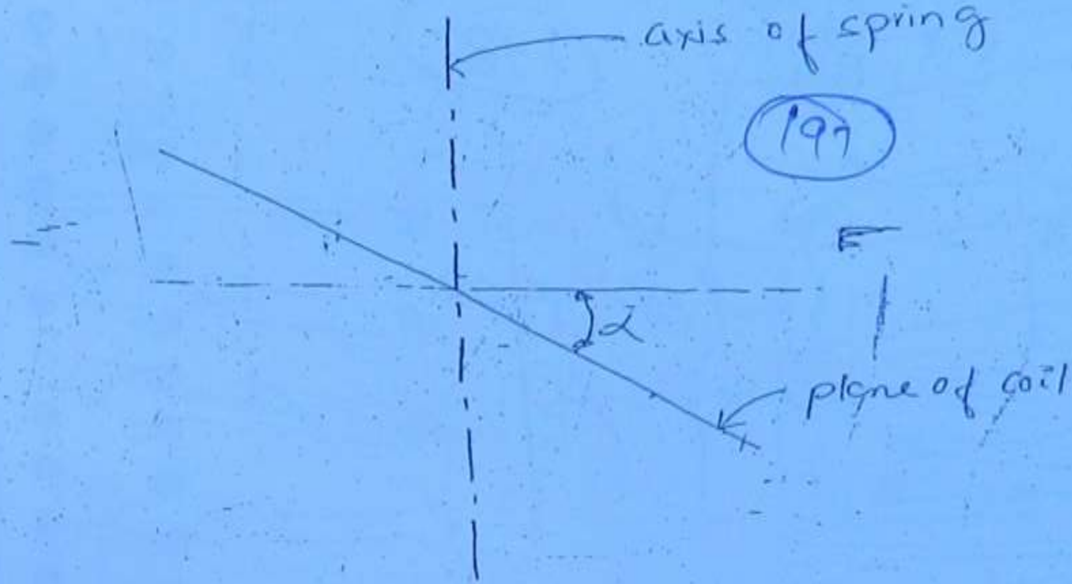
## Characteristic of an Antifriction Bearing



| Ball Bearing | Roller bearing |
| --- | --- |
| DGBB → $F_r \uparrow, F_a \downarrow$ i.e., $\frac{F_r}{F_a} > 1$ | (i) Cylindrical Roller bearing: only $F_r$ i.e., ($F_a \approx 0$) |
| • Noise is less. | (ii) Tappered Roller bearing: It withstand high Radial loads, and high thrust loads. |
| Self aligning BB: They permit some amount of angular misalignment between shaft and bearing axes due to it its self aligning properties | (iii) Spherical roller bearing: It has self aligning properties, both $F_r$ and $F_a$ |
| ACBB (angular contact): $F_r \downarrow, F_a \uparrow$ | (iv) Needle Roller bearing: • where Radial space is a constraint. • No cages • used in oscillating motions |
| i) Single row ACBB: They withstand ($F_a$) thrust loads only in one direction | (v) Thrust Roller bearing: • They withstand only axial thrust ($F_a$). |
| ii) double row ACBB: $F_a$ in both direction. | |
| Thrust bearing: They withstand $F_a$ | |

## ⑨ SPRINGS

Helical compression springs [close coiled]




$\alpha = 0 \Rightarrow$ plane of coil is at $\perp$ar to axis of spring

close coiled $\Rightarrow$ $\alpha = 7$ to $8^\circ$

ie, plane of coil are almost at right angle to axis of the spring.

$C = 1 \longrightarrow d = 50 \qquad D = 50$

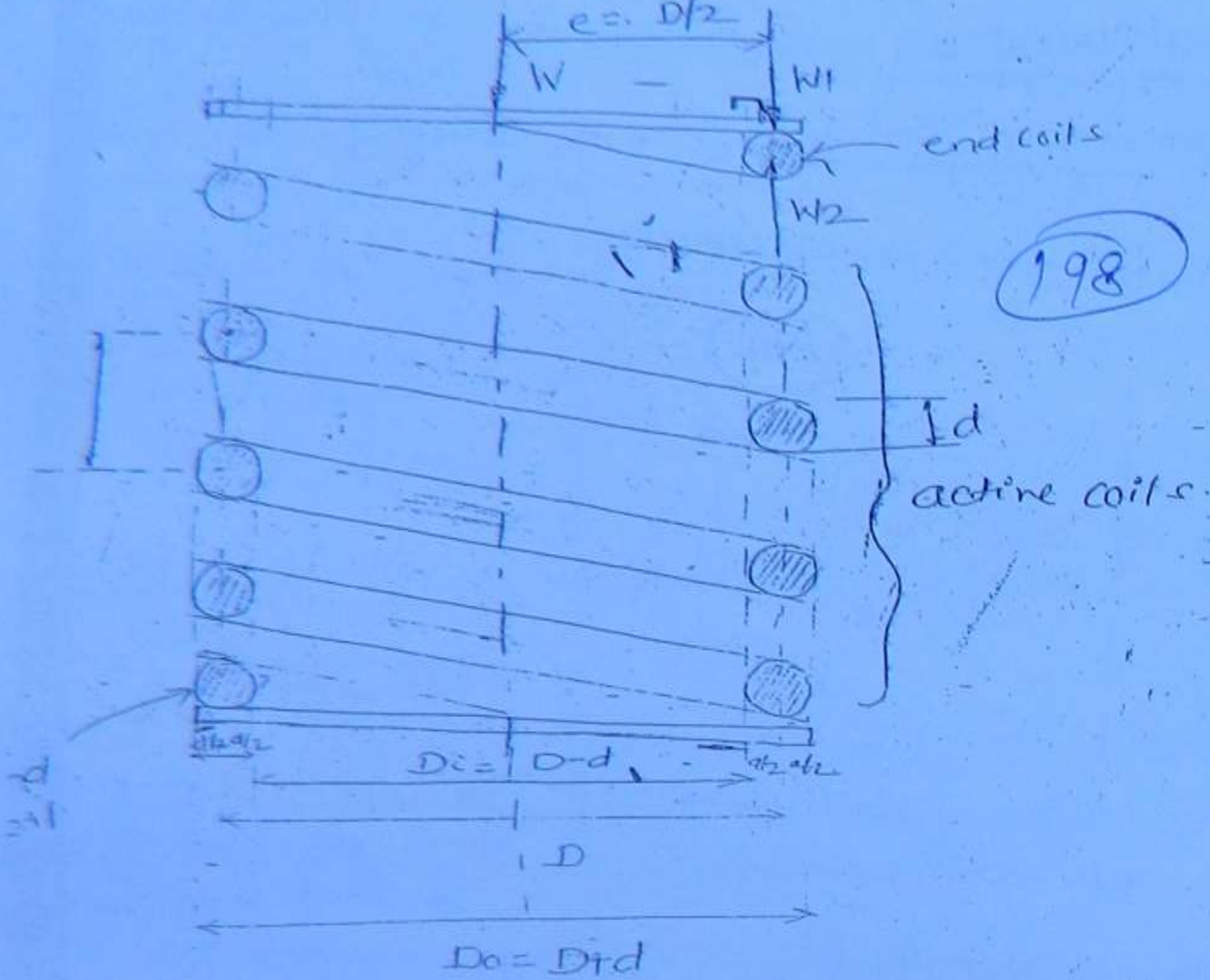
$C = 4 \Rightarrow d = 50 \qquad D = 200$

$C = 20 \Rightarrow d = 50 \qquad D = 1000$

$$C = \frac{D}{d}$$

$\therefore C \nless 3$

$\ngtr 12$




d = dia of spring wire

D = Dia of spring

= mean coil diameter

Do = outer dia of spring = D+d

Di = Inner dia of spring = D-d

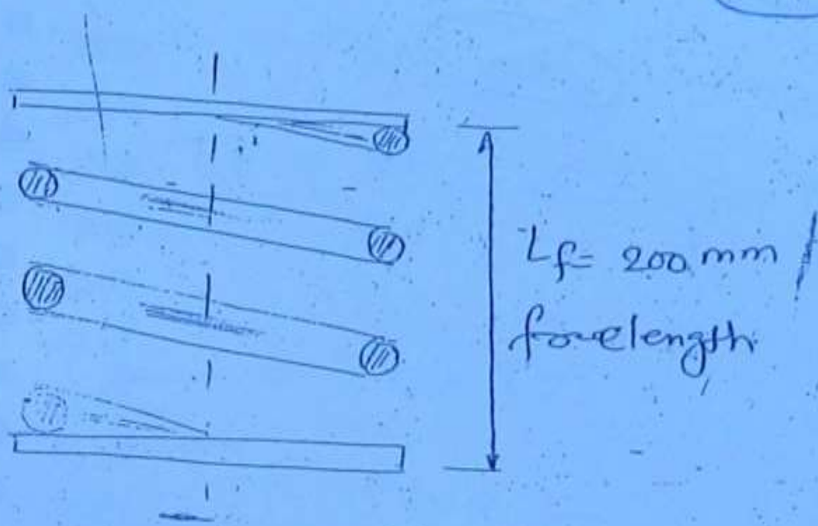
$$\boxed{C = \text{Spring Index} = \frac{D}{d}}$$

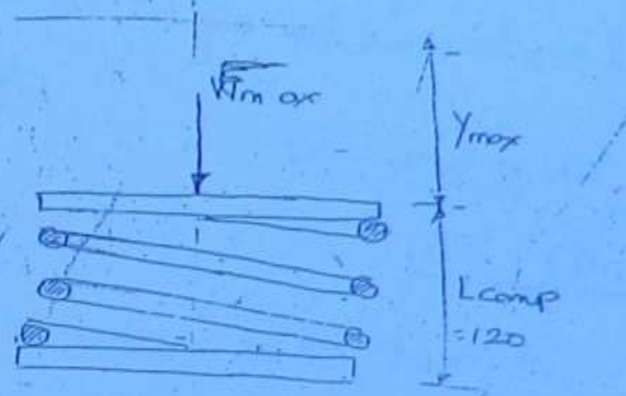
$\Rightarrow$ C = 4 to 12

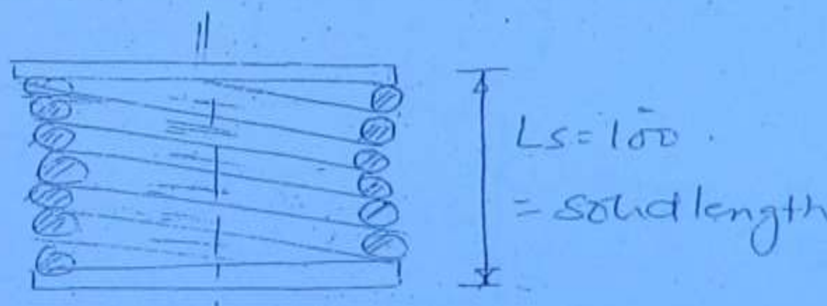
K = Spring stiffness = Spring rate

$$K = \frac{-W}{y} \text{ or } \frac{\Delta W}{\Delta y}$$






$L_{comp}$ = $L_s$ + Total gap between coils under maximum deflected position

$L_f = L_{comp} + Y_{max}$

= $L_s$ + total gap between coil under maximum deflection position + $Y_{max}$

= $L_s$ + 15% of $Y_{max}$ + $Y_{max}$

$$\boxed{L_f = L_s + 1.15\,Y_{max}}$$

Solid Length $= L_f = n.d + 1.15\, y_{max}$

$$\boxed{L_s = n \cdot d}$$

200

$n$ = no. of active coils.
$d$ = diameter of wire.

Now $W_1 = W_2 = W$.

<u>Effect of $W_1$:</u>

$\tau_1$ = direct shear stress

$$\tau_1 = \frac{W_1}{\frac{\pi}{4} d^2}$$

$$\tau_1 = \frac{4W}{\pi d^2} \longrightarrow (1)$$

<u>Effect of $W$ and $W_2$</u>

It causes twisting moment

$$TM = W \times \frac{D}{2}$$

$$\tau_2 = \frac{T}{Z_P} = \frac{16T}{\pi d^3} = \frac{\cancel{16}^{8} \times W \frac{D}{\cancel{2}}}{\pi d^3}$$

$$\tau_2 = \frac{8WD}{\pi d^3} \longrightarrow (2)$$

Now

$$\tau_1 = \frac{4W}{\pi d^2} \times \frac{2D}{d} \times \frac{d}{2D}$$

$$= \frac{8WD}{\pi d^3} \times \frac{d}{2D}$$

201

$$\therefore \tau_1 = \frac{8WD}{\pi d^3} \times \frac{0.5}{C} \rightarrow (B)$$

Now

$$\tau_{max} \text{ or Resultant} = \tau_1 + \tau_2$$

$$= \frac{8WD}{\pi d^3}\left[1 + \frac{0.5}{C}\right]$$

$$\boxed{\tau_{max} = \tau_R = \frac{8WD}{\pi d^3} \times K_{sh}}$$

$K_{sh}$ = shear stress correction factor

$$\boxed{K_{sh} = 1 + \frac{0.5}{C}}$$

$$\tau_{max} = \tau_R \times K_C$$

$$\therefore \tau_{max} = \frac{8WD}{\pi d^3} \times K_{sh} \times K_C$$

w = wahl's factor

$K_w = K_{sh} \cdot K_c$ ← curvature effect

$$\therefore \boxed{K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}}$$

2.02

$$\boxed{(\tau_{max})_{ind} = \frac{8WD}{\pi d^3} \times K_w}$$

$$\therefore \text{ or } (\tau_{max})_{ind} = \frac{8WC}{\pi d^2} \times K_w \leq \tau_{per}$$

$\therefore$ $d \geq$ min can be found.

$$\therefore \boxed{(\tau_{max})_{ind} = \frac{8 W_{max} \cdot C}{\pi d^2} K_w \leq \tau_{per}}$$

$D = Cd$

$D_o = D + d$

$D_i = D - d$

above parameters can be calculated by calculating 'd'

## Expression of $Y_{max}$

Using the strain energy stored in the spring due to twisting we can find $Y_{max}$.

$U$ = SE stored in spring

203

$$U = \frac{1}{2} T \cdot \theta$$

$$U = \frac{1}{2} \times T \times \frac{TL}{GJ} = \frac{T^2 L}{2GJ}$$

$$T = WD/2 \quad , L = \pi D n$$

$$U = \left(\frac{WD}{2}\right)^2 \times \frac{\pi D n}{2 \times G \times \frac{\pi}{32} d^4}$$

$$U = \frac{W^2 D^3 n \, 32}{8 G d^4}$$

$$U = \frac{4 W^2 D^3 n}{G d^4}$$

by using Castiglianos theorem

$$y_{max} = \frac{\partial u}{\partial p}$$

$$y_{max} = \frac{8 W D^3 n}{G d^4}$$

$$\boxed{y_{max} = \frac{8 W_{max} C^3 n}{G d}}$$

∴ from above eqn 'n' can be determined.

(204)

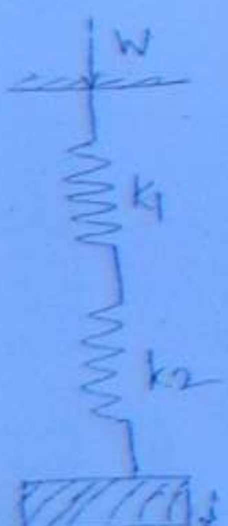
Now $k = \frac{W_{max}}{y_{max}}$

$$\boxed{k = \frac{Gd}{8C^3 n}}$$

$\Rightarrow$ $k \propto \frac{1}{n}$

When the spring is cut into 'n' equal parts the stiffness of the spring increases by 'n' times

Series



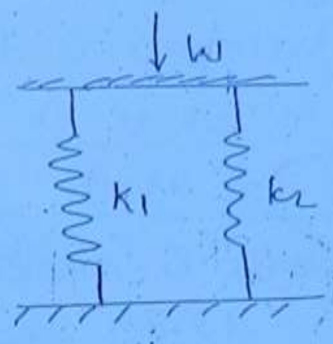

$$W_1 = W_2 = W$$

$$y = y_1 + y_2$$

$$\frac{W}{k} = \frac{W_1}{k_1} + \frac{W_2}{k_2}$$

$$\boxed{\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}}$$

Parallel connection




205

$$y_1 = y_2 = y$$

$$W = W_1 + W_2$$

$$\therefore ky = k_1 y_1 + k_2 y_2$$

$$\boxed{k_{eq} = k_1 + k_2}$$

1) diameter of spring wire (d)

$$(\tau_{max})_{ind} \le \tau_{per}$$

$$\frac{8 \cdot W_{max} \cdot C}{\pi d^2} \cdot K_w \le \tau_{per}$$

$$\therefore d \ge \text{—} \ \text{mm}$$

2) dimensions of springs

$$D = Cd$$

$$D_o = D + d$$

$$D_i = D - d$$

3) No. of active coils (n)

$$Y_{ind} = \frac{8 W C^3 n}{G d} \le Y_{per}$$

or

$$\ge Y_{per}$$

$\therefore n \leq \frac{?}{}$

$n \geq \frac{?}{}$ can be found out.

4) Find out $y_{max} = ?$

206

$$y_{max} = \frac{8 W_{max} C^3 n}{G d}$$

(5) Solid length ($L_s$)

$$L_s = n \cdot d$$

(6) free length ($L_f$)

$$L_f = L_s + 1.15\, y_{max}$$

(7) check for buckling

$$\frac{L_f}{D} \leq 3.5 \Rightarrow \text{No buckling}$$

$$\frac{L_f}{D} > 3.5 \Rightarrow \text{buckling occurs}$$

~~When~~ preventing buckling ~~occurs then the~~ of the springs, the spring should be placed in a sleeve or a hollow cylinder.

# (10) CLUTCHES



clutch is defined as a mechanical device which is used to transmit power from a driver shaft to a driven shaft.

⇒ clutch is also defined as a mechanical device which is used to engage/disengage the driven shaft to/from the driver shaft at the will of the operator.

⇒ The main function of clutch is to avoid frequent stopping or starting of the prime mover i.e, by means of clutch the vehicle can be stop or started any number of times, without stopping the prime mover or engine.

- clutches
  - Jaw clutches (+ve drive) eg: gears
  - friction clutches (slip drive)
    - axial Friction clutches
      - plate clutches
        - Single plate clutch
        - multi plate clutch
      - cone clutches
    - Radial friction clutches
      - Centrifugal clutches

xial friction clutches: force/pressure is applied along the shaft axis. (208)

adial friction clutches: forces is applied in the radial e (⊥ar to the shaft axis)

⇒SPC used in 4 wheeler like Tractors, Trucks!

⇒MPC used where space is a problem like bikes, scooters. Autos.

Centrifugal clutches (Automatic clutches): used in mopeds.

Jaw clutches: used in M/c tools Rolling mills.

| Old or worn out clutches (UWT) | New clutches (UPT) |
| --- | --- |
| $T = \frac{P \times 60}{2\pi N} \times 10^6 = ?\ Nm$ | T. Same |
| $T_f = n \frac{1}{2} \mu W [R_o + R_i]$ | $T_f = n \times \frac{2}{3} \mu W \left[ \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} \right]$ |
| W = axial force reqd to engage the clutch. | $W = p \times \pi (R_o^2 - R_i^2)$ |

Old or worn out clutches (UWT)

New clutches (UPT)

209

$W = p \times 2\pi R_i (R_o - R_i)$

$T_f = n \times \frac{1}{2} \mu \times p_i \times 2\pi R_i (R_o^2 - R_i^2)$

$\Rightarrow T_f = n \cdot \mu \cdot p \cdot \pi R_i (R_o^2 - R_i^2)$

So, $R_o = ?$

$\Rightarrow n =$ no. of pair of contact surfaces.

$n = 1 \Rightarrow$ SPC

$n = 2 \Rightarrow$ SPC effective on either side

$\Rightarrow n = n_1 + n_2 - 1 \Rightarrow$ MPC

$T_f = n \times \frac{2}{3} \mu p \pi (R_o^3 - R_i^3)$

$T_f = \frac{2}{3} n \mu p (R_o^3 - R_i^3)$

$\therefore R_o = ?$ can be calculated

where $n_1 =$ no. of plates or discs attached to a driver shaft

$n_2 =$ no. of plates or disc attached to a driven shaft

$n =$ even number in Multiplate plate clutch

# (11) POWER SCREWS

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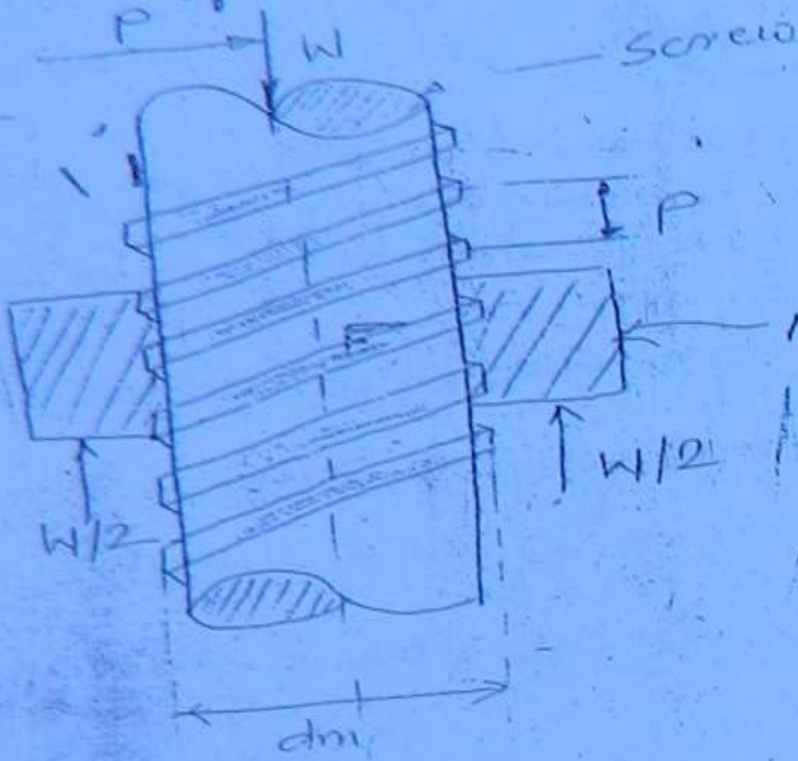


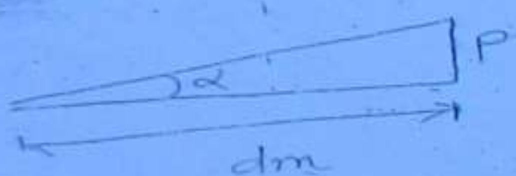


Load applied on screw through horizontal effort

Load to lifted vertically = W

Mean diameter = dm

effort applied is one revolution and load is lifted axially by pitch P of the thread for single start threads and by lead of thread for multi start threads.

Helix angle $\alpha = \tan^{-1}\left(\frac{P}{\pi d_m}\right) \rightarrow$ Single start

$\alpha = \tan^{-1}\left(\frac{Lead}{\pi d_m}\right)$ → for multi-start threads

(211)

⇒ Let $\mu$ = coefficient of friction between Screw and Nut.

Let $\phi$ = angle of friction

∴ We have $[\mu = \tan\phi]$

Effort to raise the load ($P_r$)

$P_r = W \tan(\alpha + \phi)$

$$= W \left[ \frac{\frac{lead}{\pi d_m} + \mu}{1 - \frac{lead}{\pi d_m} \times \mu} \right]$$

Turning moment applied on screw to raise the load $= T_r$

$$\boxed{T_r = W \cdot \frac{d_m}{2} \tan(\alpha + \phi)}$$

Cond 1: If $\boxed{\alpha > \phi}$

then, after the removal of the effort 'P' Load W will come down without applying any rotational moment on the nut.

$\boxed{\alpha < \phi}$ **

Then after the removal of the effort 'p' load 'W' will remain in position that is locked in the position without applying any brake and the screw is said to be self locked.

Now, for $\phi > \alpha$, then effort will be required to lower the load.

$$P_{Lower} = W \tan(\phi - \alpha)$$

Turning moment $T_L = W \frac{d_m}{2} \tan(\phi - \alpha)$

Efficiency of power screw

Ideal effort = It is that effort required to raise the load when $(\phi = 0)$

$$P_{ri} = W \tan\alpha$$

$$\therefore \eta_{power\ screw} = \frac{\text{Ideal effort}}{\text{actual effort}}$$

$$= \frac{P_{ri}}{P_{ra}}$$

$$\eta_{ps} = \frac{\cancel{W} \tan\alpha}{\cancel{W} \tan(\alpha+\phi)}$$

$$\boxed{\eta_{ps} = \frac{\tan\alpha}{\tan(\alpha+\phi)}}$$

⇒ for self locking screw i.e, $(\phi > \alpha)$

$\boxed{\text{We get } \eta_{ps} < 50\%}$ ** (for any PSU exams)

⇒ $\eta_{ps}$ (Tranezoidal thread) $<$ $\eta_{ps}$ (square thread)

Condition for Maximum efficiency:

i.e, To determine 'α'

$$\frac{d\eta}{d\alpha} = 0$$

$$\boxed{\alpha = \frac{\pi}{4} - \frac{\phi}{2}}$$ **

→ Cond$^n$ for maximum efficiency

RAVI